KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

Marks Scored:

Level: B.Sc.
Year : II

Course : STAT 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F.M. : 20

Registration No.:

Date 04 JUN 2019

SECTION "A"

[10Q. × 1 =10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. If two variables oppose each other then the correlation will be..............

2. The joint probability mass function of (X, Y) is given by p(x, y) = k (4x+4y), x = 1, 2, 3; y = 0, 1, 2. Then the marginal of Y is ....

3. In the previous question 2, $P(X \leq 2|Y \leq 1) =$ ....

4. Suppose you have a random sample of 10 observations from a normal distribution with mean = 10 and variance = 2. The sample mean (x-bar) is 8 and the sample variance is 3. The sampling distribution of $\bar{X}$ is ....

5. The cumulative distribution function (CDF) of X is given by $F(x) = 1 - e^{-x/3}$, for x > 0 then the random variable X is distributed as...............with parameter(s)..................

6. The probability density function of the random variable described by the CDF in question (5) is ....

7. The memory less property is characterized by $P[X > s + t| X > s] =$...............

8. A continuous random variable is said to have a gamma distribution with parameters (α, λ), where $\alpha > 0$ and $\lambda > 0$, if its probability density function is given by ....

9. The moment generating function of the gamma distribution is given by $M_X(t) = (1 - \frac{t}{\lambda})^{-\alpha}$ then the mean is given by ....

10. If X is a Weibull random variable with parameters $\alpha > 0$ and $\beta > 0$, then the distribution function of X is ....

SECTION "B"

[10Q. × 1 = 10 marks]

Choose the most appropriate answer from the given options and ***encircle*** *the letter of your choice.*

11. If the covariance between two random variables X and Y is zero then ....
    a. X and Y are independent
    b. Knowing the value of X provides no information about the value of Y
    c. E(X) = E(Y) = 0
    d. none of the above

12. If two random variables X and Y are independent, then ....
   a. their joint distribution equals the product of their marginal distributions
   b. the conditional distribution of X given Y equals the marginal distribution of X
   c. their covariance is zero
   d. a, b, and c

13. ....theorem is an important result in statistics, which states that normal distribution is the limiting distribution of the sum of independent random variables with finite variance as the number of these random variables gets indefinitely large.
   a. Central Limit Theorem
   b. Lindeberg - Levy central limit theorem
   c. Chebyshev's Inequality
   d. Cauchy Schwarz Inequality

14. The relationship between price and demand of a commodity and sale of woolen garments and the day temperature are examples of ....
   a. positive correlation
   b. negative correlation
   c. zero correlation
   d. none of these

15. If the joint probability density function (pdf) of a bivariate random variable (X, Y) is given by $f(x,y) = \{2, 0 < y \leq x < 1 \text{ and } 0 \text{ otherwise}\}$. Then the marginal density function of X is ....
   a. $2x \text{ if } 0 < x < 1$
   b. $2(1-y) \text{ if } 0 < y < 1$
   c. $1/x \text{ for } 0 < y < x$
   d. $1/(1-y) \text{ for } y < x < 1$

16. In the previous question no. 15 the conditional probability of Y, given X = x is ....
   a. $2x \text{ if } 0 < x < 1$
   b. $2(1-y) \text{ if } 0 < y < 1$
   c. $1/x \text{ for } 0 < y < x$
   d. $1/(1-y)$

17. In the linear equation $Y = A + BX + CZ$, the change in Y for unit change in X keeping the effect of Z fixed is ....
   a. A
   b. B
   c. C
   d. None of the mentioned

18. If r = 0.6, byx = 1.2 then bxy=...............
   a. 0.3
   b. 0.2
   c. 0.72
   d. 0.40

19. If X and Y are independent random variables, then E(XY) is equal to ....
   a. E(X)+E(Y)
   b. X.E(Y)
   c. E(X).Y
   d. E(X)E(Y)

20. When do the conditional density functions get converted into the marginally density functions?
   a. Only if random variables exhibit statistical dependency
   b. Only if random variables exhibit statistical independency
   c. Only if random variables exhibit deviation from its mean value
   d. None of the above

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Semester : I
F. M. : 55

SECTION "C"

[3Q. × 7 = 21 marks]

1. Differentiate between univariate and bivariate random variables. Given the joint probability density function of (X, Y) as

$$f(x,y) = 8xy, 0 < y < x < 1$$
$$= 0, \text{otherwise}$$

   a. Show that X and Y are not independent
   b. Find P(X+Y>1)

2. The demand for a new product of a company is assumed to be continuous random variable with the distribution function

$$F(x) = \begin{cases} 1 - e^{-\frac{x^2}{2\alpha^2}} & if\ x>0 \\ 0 & if\ x < 0 \end{cases}$$

   Find
   a. The probability density function of X
   b. The mean and variance of X
   c. The probability that the demand of new product will exceed α

3. The data represents the age in months of children and their mean daily dietary calorie intake.

Age (P)	14	16	16	20	20	24	30	32	34	36
Mean Cal (Q)	860	867	870	887	880	900	950	950	960	980

   a. Display the data on a scatter plot.
   b. Describe the association between the two variables in terms of direction, form and strength.
   c. Fit a line of regression of Q on P

SECTION "D"

[6Q. × 4 = 24 marks]

4. Two random variables have following joint probability density function

$$f(x,y) = 2 - x - y, 0 \le x \le 1, 0 \le y \le 1$$
$$= 0, otherwise$$

   Find the marginal density functions of X and Y.

5. If X is gamma random variable with probability density function

$$f(x) = \begin{cases} \frac{\lambda}{\Gamma\alpha}(\lambda x)^{\alpha-1}e^{-\lambda x} & for\ x > 0 \\ 0 & elsewhere \end{cases}$$

   Find the mean and variance

6. The fraction X of male runners and the fraction Y of female runners who compete marathon races can be described by the joint density function:

$$f(x,y) = 8xy, 0 \leq x \leq 1, 0 \leq y \leq x$$
$$= 0, otherwise$$

Find the covariance of X and Y.

7. Consider the probability distribution of the discrete random vector [X1, X2] where X1 represents the number of orders for Aspirin in August in neighborhood drugstore and X2 represents the number of orders in September.
   a. Find the marginal distributions.
   b. Find the Expected sales in September given that the sales in August were 51, 52, 53, 54 and 55.

X_2 \ X_1	51	52	53	54	55
51	0.06	0.05	0.05	0.01	0.01
52	0.07	0.05	0.01	0.01	0.01
53	0.05	0.10	0.10	0.05	0.05
54	0.05	0.02	0.01	0.01	0.03
55	0.05	0.06	0.05	0.01	0.03

8. If X is a random variable with probability density function

$$f(x) = \begin{cases} 2xe^{-x^2} & if\ x > 0 \\ 0 & otherwise \end{cases}$$

And $Y = X^2$, find the PDF of Y using distribution function method and transformation methods. Also validate the obtained results.

9. Let X and Y be two random variables taking three values 0, 1 and having the joint probability distribution:

Y	0	1
X		
0	0.4	0.1
1	0.2	0.3

Find the correlation of X and Y.

SECTION "E"

[5Q. × 2 = 10 marks]

10. If $y = 2x - 3$ and $y = 5x + 7$ are two regression lines, find
    (i) the mean values of X and Y
    (ii) the correlation coefficients between X and Y.

11. Verify Cauchy Schwarz inequality for data in Question No. 9.

12. Given the random variable X with probability density function $f(x) = 2x$, if $0 < x < 1$
    Find the probability density function of $Y = 8X^3$.

13. A random sample of size 200 is taken from a population whose mean is 50 and variance is 600. Using central limit theorem, find the probability that the mean of the sample will not differ from $\mu = 50$ by more than 5.

14. If X is a random variable with Cumulative Distribution Function as F(x), show that the random variable Y = F(X) is uniformly distributed in (0, 1).