KATHMANDU UNIVERSITY
End Semester Examination [C]
June, 2018

Marks Scored:

Level : B. Sc.
Year : II

Course : STAT 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : JUN 17 2018

SECTION "A"
[10 Q. × 1 =10 marks]

Fill in the blanks space*(s)* by most appropriate word*(s)* or symbol*(s)*.

1. The cumulative distribution function (CDF) of X is given by $F(x) = 1 - e^{-\frac{x}{2}}$ for $x > 0$ then the random variable X is distributed as.....with parameter(s).....

2. The probability density function of the random variables described by CDF in Question (1) is .....

3. The memory less property characterized by P[X>s+t|X>s] =.....

4. Two discrete random variables X and Y have P[X=0, Y=0] = 2/9, P[X=0, Y=1]=1/9, P[X=1, Y=0]= 1/9, P[X=1, Y=1]=5/9 then P[X=0|Y=0]=.....

5. The moment generating function of Gamma distribution is given by $M_X(t) = (1 - \frac{t}{\lambda})^{-\alpha}$ Then the mean of this distribution is given by .....

6. If the Cumulative distribution function of X is F(x) then the Cumulative Distribution Function of $Y = X^3$, G(y) is .....

7. Let X be a random variable denoting the number of time face six appears in a single roll of a die. Then the probability distribution of X can be completed in the table as.....

X	.....	.....
P(X)	.....	.....

8. The probability distribution of $Y = (X - 1)^2$ is to be filled up in the following table. Here X is the random variable mentioned in problem 7.

Y	.....	.....
P(Y)	.....	.....

9. If two lines of regression are given as X+2Y-5=0 and 2X+3Y=8, then the mean value of X and Y are respectively .....

10. The regression lines Y on X and X on Y are respectively Y = a x + b and X = c y + d then correlation coefficient between X and Y is.....

SECTION: "B"

[10 Q. ×1 = 10 marks]

11. .....theorem is an important result in statistics, which states that normal distribution is the limiting distribution of the sum of independent random variables with finite variance as the number of these random variables gets indefinitely large.

a) Central Limit Theorem
b) Lindeberg - Levy Theorem
c) Chebyshev's Inequality
d) Cauchy Schwartz Inequality

12. If the joint probability density function of a bivariate random variable (X, Y) is given by $f(x,y) = 2, 0 < y \leq x < 1$ *and* 0 *otherwise* then the marginal density function of X is .....

a) $2x \; if \; 0 < X < 1$
b) $2(1-y) \; if \; 0 < y < 1$
c) $\frac{1}{x} \; for \; 0 < y < x$
d) $\frac{1}{1-y} \; for \; y < x < 1$

13. In the previous question no 12, the conditional density of Y, given X = x is.....

a) $2x \; if \; 0 < x < 1$
b) $2(1-y) if \; 0 < y < 1$
c) $\frac{1}{x} \; for \; 0 < y < x$
d) $\frac{1}{1-y} \; for \; y < x < 1$

14. The regression coefficients are b_1 and b_2. Then the correlation coefficient r is .....

a) $\frac{b_1}{b_2}$
b) $\frac{b_2}{b_1}$
c) $b_1 b_2$
d) $\pm\sqrt{b_1 b_2}$

15. If the cumulative distribution function of X is F(x) then the cumulative distribution function G(y) of Y = X + a is .....

a) F(y-a)
b) F(y+a)
c) $F(y^{1/3})$
d) $F(\frac{y}{a})$

16. Suppose the regression equation for predicting Y from X is given by Y=20+5X. Then all the following are true except.....

a) the intercept is 20
b) the slope of the regression line is positive
c) the predicted value of Y for X=4 is 40
d) the correlation between X and Y is negative

17. The normal equation corresponding to X_3 on X_1 and X_2 in the linear regression $X_3 = A + BX_1 + CX_2$ is not .....

a) $\sum X_3 = A + B\sum X_1 + C\sum X_2$
b) $\sum X_1X_3 = A\sum X_1 + B\sum X_1^2 + C\sum X_1X_2$
c) $\sum X_2X_3 = A\sum X_2 + B\sum X_1X_2 + C\sum X_2^2$
d) $\sum X_3 = nA + B\sum X_1 + C\sum X_2$

18. Let (X, Y) be a two dimensional random variable. The line of regression of Y on X is.....

a) $Y - E(Y) = \frac{cov(X,Y)}{V(X)}(X - E(X))$
b) $X - E(X) = \frac{cov(X,Y)}{V(Y)}(Y - E(Y))$
c) $Y = E(Y|X = a)$
d) $X = E(X|Y = y)$

19. If X and Y are perfectly correlated that is $\rho = \pm 1$ then the regression lines.....

a) are perpendicular to each other
b) Coincide
c) are Parallel
d) are different

20. For a distribution for two dimensional random variables (X, Y) following a continuous or discrete distribution, the Cumulative Distribution Function F(x, y) is defined as .....

a) $P(X = x, Y = y)$
b) $P(X \leq x, Y \leq y)$
c) $P(X < x, Y < y))$
d) $P(X \geq x, Y \geq y)$

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Time : 2 hrs.30 mins.

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F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. The joint probability distribution of X and Y is given by [4+3]

X\Y	0	1	2
0	1/5	1/10	1/20
1	1/10	1/20	1/10
2	¼	1/20	1/10

a) Verify Cauchy Schwarz inequality for (X, Y)
b) Find E(X|Y=1)

2. Differentiate between univariate and bivariate random variables. Given the joint probability density function of (X,Y) as [3+4]

$f(x; y) = 8xy;\ 0 < y < x < 1$
$= 0;$ *otherwise*

a) Show that X and Y are not independent
b) Find P(X+Y>1)

3. What is exponential distribution? Let X represent the lifetime of a component in a piece of lab equipment and suppose that X is exponentially distributed with a mean of μ= 1000 hours.

a) Find $P(X < 1000)$
b) Find $P(50 < X < 250)$
c) Find the 90 th percentile of X
d) Why might this information be important to the lab personnel?

SECTION "D"
[6Q. × 4 = 24 marks]

4. If X_1 and X_2 are independent Poisson variables with parameters $\lambda_1$and λ_2, show that the conditional distribution of X_1 given $X_1 + X_2$ follows binomial distribution.

5. Let X and Y be two random variables taking three values -1, 0 and 1, and having the joint probability distribution

Y X	-1	0	1
-1	2/16	1/16	2/16
0	2/16	2/16	2/16
1	2/16	1/16	2/16

Find COV(X, Y) and interpret the result [3+1]

6. Construct the line of regression Y on X for the problem given in Question 5

OR

If X is a random variable with the exponential density $f(x) = \begin{cases} \theta e^{-\theta x} \text{ for } x > 0 \\ 0 \text{ elsewhere} \end{cases}$ and $Y = X^2$, find i) the distribution function of Y ii) the probability density of Y.

7. In an experiment a vertical spring fixed at its upper end was stretched by application of weights to its lower end and the length of the spring was measured. The following readings were obtained.

Load (kg)	0	1	2	3	4	5	6	7	8
Length(cm)	12	12.5	13.1	13.8	14.5	15.2	15.7	16.2	16.7

Plot these pairs of values as points on a graph and draw the straight lines which best fits them. Assuming that the relation between the Load and the Length continues to hold for greater loads, find the length of the spring when the load is 12 kg.

8. If X and Y are independent Poisson random variables with parameters λ_1 and λ_2, respectively. What is the distribution of $Z = X + Y$.

9. The joint density function of X and Y is given by

$$f(x,y) = e^{-(x+y)}, \quad x > 0, y > 0$$

Find the Probability Density Function of $U = \frac{X+Y}{2}$. Are X and Y independent?

SECTION "E"

[5Q. × 2 = 10 marks]

10. The joint density function of two continuous random variables X and Y is

$$f(x,y) = \begin{cases} Cxy, 0 < x < 4, 1 < y < 5 \\ 0, otherwise \end{cases}$$

Find the value of C.

11. If X is a random variable with Cumulative Distribution Function as F(x), show that the random variable $Y = F(X)$ is uniformly distributed in (0, 1).

12. Let X and Y be independent random variables, then prove that cov(X, Y) = 0.

13. Two random variables X and Y have the joint density function

$$f(x,y) = \begin{cases} 25, & .95 < x < 1.15, .95 < y < 1.15 \\ & 0, \text{otherwise} \end{cases}$$

Find P (XY<1)

14. Find the moment generating function of an exponential random variable with parameter λ