KATHMANDU UNIVERSITY
End Semester Examination [C]
November, 2018

Marks Scored:

Level : B.Sc.
Year : II

Course : PHYS 213
Semester: II

Exam Roll No.:

Time: 30 mins.

F.M. : 20

Registration No.:

Date : NOV 16 2018

SECTION "A"
[20 Q.×1=20 marks]

Choose and tick the most appropriate answer.
Symbols have their usual meanings unless stated and missing parameters can be assumed suitably.

1. Which one of the following statements is CORRECT?
   [a] The constraint involved in the motion of rigid body is **nonholonomic**.
   [b] The walls of a gas container constitute a **holonomic** constraint.
   [c] A mechanical system is **rheonomous** if its equations of constraints contain the time as an explicit variable.
   [d] The constraint involved on a bead sliding on a moving wire is **scleronomous**.

2. The Lagrangian for a system is $L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$, where x is a generalized coordinate, and m and k are constants. The equation of motion for this system is
   [a] $m\ddot{x}+kx=0$ [b] $m\dot{x}+kx=0$
   [c] $m\ddot{x}+k\dot{x}=0$ [d] $mx+kx=0$

3. Principle of virtual work is:
   [a] $\sum_i \vec{F}_i^{(a)}\cdot\delta\vec{r}_i=0$. [b] $\sum_i\left(\vec{F}_i^{(a)}-\dot{\vec{p}}_i\right)\cdot\delta\vec{r}_i=0$
   [c] $\sum_i\left(\vec{F}_i^{(a)}-\vec{p}_i\right)\cdot\delta\vec{r}_i=0$. [d] $\sum_i\left(\vec{F}_i^{(a)}-\dot{\vec{p}}_i\right)\cdot\delta\vec{r}_i=Q_j$

4. A particle moves in a plane under the influence of a force, acting toward a centre of force, whose magnitude is $F=\frac{1}{r^2}-\frac{\dot{r}^2}{r^2c^2}+\frac{2\ddot{r}}{rc^2}$, where r is the distance of the particle to the centre of force. The generalized potential that will result in such a force is
   [a] $U=\frac{1}{r}+\frac{\dot{r}^2}{c^2}$ [b] $U=\frac{1}{r}+\frac{\dot{r}}{rc^2}$ [c] $U=\frac{1}{r}+\frac{\dot{r}^2}{c^2}$ [d] $U=\frac{1}{r}+\frac{\dot{r}^2}{rc^2}$

5. Which one of the following statements is NOT CORRECT?
   [a] Central force motion is always motion in a plane.
   [b] The areas swept out by the radius vector drawn from the centre of the Sun to the centre of the planet in equal intervals of time are equal.
   [c] The planet moves around the sun in an elliptic orbit with the Sun at one of the foci of the orbit.
   [d] The square of the period of revolution of the planet is directly proportion to the square of the semi-major axis of the elliptic orbit.

6. The Figure A-2 represents the orbit of a planet around a star, S, and the marks divide the orbit into 14 equal time intervals, $t = \frac{T}{14}$, where T is the orbital period. If the only force acting on the planet is Newtonian gravitation, then true statements about the situation include which of the following?

I. Area A= area B

II.

III. The star S is at one focus of an elliptically shaped orbit.

IV. $T^2 = C\,a^3$, where a is the semi-major axis of the ellipse and C is a constant

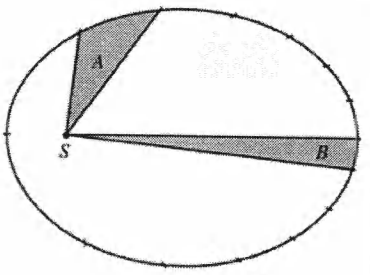

Figure A-2

[a] I only [b] II only [c] I and III only [d] I,II, and III

7. If a particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle, then the force varies as the
[a] inverse-fourth power of the distance.
[b] inverse-fifth power of the distance.
[c] inverse-square of the distance.
[d] fifth power of the distance.

8. The ratio of maximum and minimum speeds of Earth, taking the eccentricity of the Earth's orbit to be 0.023 is
[a] 0.047 [b] 1.047 [c] 0.995 [d] 0.0167

9. The coordinate system $C' \equiv O\xi\eta\zeta$ is obtained by rotating the coordinate system $C \equiv Oxyz$ through an angle of 30^0 about the axis Oz. The transformation matrix between C and C' is

[a] $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{pmatrix}$ [b] $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 \\ 0 & -1/2 & \sqrt{3}/2 \end{pmatrix}$

[c] $\begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ [d] $\begin{pmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

10. The law of conservation of momentum applies to a collision between two bodies if
[a] they exert equal and opposite forces on each other.
[b] they exert forces on each other respectively proportional to their masses.
[c] they exert forces on each other respectively proportional to their velocities.
[d] their accelerations are proportional to their masses.

11. A 'm' kg object moving 'v' m/s collides with and sticks to an 'M' kg object initially at rest. The kinetic energy lost by the system as a result of this collision.

[a] $\frac{1}{2}\left[\frac{mM}{m+M}\right]v^2$ [b] $\frac{1}{2}\left[\frac{m+M}{mM}\right]v^2$ [c] $\left[\frac{mM}{m+M}\right]v^2$ [d] $\frac{1}{2}\left[\frac{mM}{m+M}\right]v$

12. Which one is variant under Galilean transformations?
[a] Velocity.
[b] Newton's laws of motion.
[c] The law of conservation of momentum.
[d] The law of conservation of energy.

13. The number of degrees of freedom for the general motion of a rigid body in space is
[a] 1 [b] 2 [c] 3 [d] 6

14. The Hamiltonian corresponding to the Lagrangian $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ is

[a] $\frac{p_x^2}{4a^2} + \frac{p_y^2}{4b^2} + kxy$

[b] $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - kxy$

[c] $\frac{p_x^2 + p_y^2}{4ab} + kxy$

[d] $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$

15. A mass attached to the end of a massless rod of length L is free to swing below the plane of support, as shown in Figure A-2. The Hamiltonian for this system is given by

$$H = \frac{p_\theta^2}{2mL^2} + \frac{p_\phi^2}{2mL^2 \sin^2\theta} - mgL\cos\theta$$

where θ and ϕ are defined as shown in the Figure A-2. On the basis of Hamilton's equations of motion, the generalized coordinate or momentum that is a constant in time is

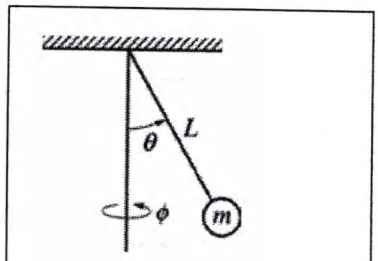

Figure A-2

[a] θ [b] ϕ [c] p_ϕ [d] p_θ

Fill the following blanks with appropriate answers.

16. The Lagrangian for a mechanical system is $\boxed{L = a\dot{q}^2 + bq^4}$, where q is a generalized coordinate and a and b are constants. The equation of motion for this system is.....

17. The reduced mass of Hydrogen molecule is nearly equal to the mass of .....

18. The ..... is a phenomenon, related to the Earth's rotation that causes air and ocean currents to deflect to the right to their direction of motion in the northern hemisphere and to the left of their direction of motion in the southern hemisphere.

19. The moment of inertia tensor is symmetric and it has only .....independent components.

20. The Hamiltonian corresponding to the Lagrangian $\boxed{L = \frac{\dot{q}^2}{4} - \frac{q^2}{9}}$ is .....

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SECTION "B"
[5 Q.×4=20 marks]

1. What are generalized coordinates and cyclic coordinates? Show that generalized momentum conjugate to a cyclic coordinate is conserved.

OR

Set up the Lagrangian and obtain Lagrange's equation for a simple pendulum. Deduce the formula for its time period.

2. State Kepler's laws of planetary motion. Prove that the radius vector sweeps out equal areas in equal times.

3. What are Galilean Transformations? Show that the laws of conservation of momentum and energy are invariant to Galilean transformations.

4. Explain briefly the centre-of-mass system. If θ_1 and θ are angles of scattering in the laboratory systems and centre-of-mass system respectively, prove that $\boxed{\tan\theta_1 = \dfrac{\sin\theta}{\cos\theta + \dfrac{m_2}{m_2}}}$.

OR

Suppose that the moments and products of inertia of a rigid body R with respect to an xyz coordinate system intersecting at origin O are $I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}$ respectively. Prove that the moment of inertia of R about an axis making angles α, β, γ with the x, y and z axes respectively is given by

$$\boxed{I = I_{xx}\cos^2\alpha + I_{yy}\cos^2\beta + I_{zz}\cos^2\gamma + 2I_{xy}\cos\alpha\cos\beta + I_{xz}\cos\alpha\cos\gamma + I_{yz}\cos\beta\cos\gamma}.$$

5. What is the Hamiltonian function? Derive the canonical equations of Hamilton.

SECTION "C"
[5 Q.×7=35 marks]

6. Explain D'Alembert's principle. Derive Lagrange's equations of motion for a conservative standard system.

7. Derive the differential equation of an orbit in polar coordinates under central force. If the equation of an orbit of a particle under the action of the central force is given by $\boxed{r = \dfrac{p}{1+\varepsilon\cos\theta}}$ where p and ε are constant, then show that the corresponding force is given by $\boxed{f(r) = -\dfrac{l^2}{mp}\dfrac{1}{r^2}}$.

8. Obtain Euler's equations of motion for a rotating rigid body with a fixed point. From Euler's equations of motion for a rigid body, having no external torque about a fixed point, prove that $\boxed{T=\frac{1}{2}I_1\omega_1^{\,2}+\frac{1}{2}I_2\omega_2^{\,2}+\frac{1}{2}I_3\omega_3^{\,2}=\text{constant}}$, and $\boxed{L=I_1\omega_1\hat{i}+I_2\omega_2\hat{j}+I_3\omega_3\hat{k}=\text{constant}}$, where the terms have standard meaning.

OR

Define a rigid body. Derive an expression for the angular momentum and the rotational kinetic energy of a rigid body.

9. What is a collision? Explain briefly elastic and inelastic collision. A particle of mass m_1 suffers perfectly elastic collision with another particle of mass m_2 at rest in the laboratory frame of reference. After scattering m_1 and m_2 move at angles θ_1 and θ_2 with respect to the original direction of m_1. Discuss the elastic collision between the two particles in the lab system. Show that in the lab system the particles of the same mass will be at right angles to each other after collision.

OR

Show that the differential scattering cross-section for scattering of α-particles by an atomic nucleus is given by

$$\boxed{\sigma(\Theta)=\frac{1}{4}\left[\frac{ZZ'e^2}{2E}\right]^2 \text{cosec}^4\frac{\Theta}{2}}.$$

where the symbols have their usual meaning.

10. Write down the Hamiltonian for a simple pendulum and a compound pendulum.

Consider a Lagrangian of the form $\boxed{L=\frac{1}{2}m(\dot{x}^2-\omega^2x^2)\,e^{\gamma t}}$, where the particle of mass m moves in one direction. Assume all constants are positive.

(a) Find the canonical momentum and construct the Hamiltonian. Is this Hamiltonian a constant of motion?

(b) Find the equations of motion.