KATHMANDU UNIVERSITY
End Semester Examination
August/September 2017

AUG 2 8 2017

Level : B. Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : PHYS 213
Semester : II
F. M. : 55

SECTION "B"
[5Q × 4 = 20 marks]

1. A particle moves in a plane under the influence of a force, acting toward a centre of force, whose magnitude is

$$F = \frac{1}{r^2}\left(1 - \frac{\dot{r}^2 - 2\ddot{r}\,r}{c^2}\right),$$

where r is the distance of the particle to the centre of force. Find the generalized potential that will result in such a force and from that the Lagrangian for the motion in a plane.

OR

Set up Lagrangian equation for an Atwood's machine and find an expression for its acceleration.

2. A reference frame 'r' rotates with respect to inertial reference frame 's' with the (constant) angular velocity $\vec{\omega}$. If the radius vector, velocity and acceleration of a particle of mass m in frame 'r' are represented by $\vec{r}$, $\vec{v}_r$ and $\vec{a}_r$ respectively, show that the equation of motion in the inertial system is given by $\vec{F} - 2m(\vec{\omega} \times \vec{v}_r) - m\omega \times (\vec{\omega} \times \vec{r}) = ma_r$.

3. Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle, then the force varies as the inverse-fifth power of the distance.

OR

Prove Kepler's third law: The square of the periods of the various planets are proportional to the cube of their respective semi major axes with different proportionality constant.

4. What is the Hamiltonian function? Derive the canonical equations of Hamilton.

5. Suppose that the moments and products of inertia of a rigid body R with respect to an xyz coordinate system intersecting at origin O are $I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}$ respectively. Prove that the moment of inertia of R about an axis making angles α, β, γ with the x, y and z axes respectively is given by

$$I = I_{xx}\cos^2\alpha + I_{yy}\cos^2\beta + I_{zz}\cos^2\gamma$$
$$+ 2I_{xy}\cos\alpha\cos\beta + I_{xz}\cos\alpha\cos\gamma + I_{yz}\cos\beta\cos\gamma.$$

SECTION "C"
[5Q × 7 = 35 marks]

6. Obtain Euler's equations of motion for a rotating rigid body with a fixed point. From Euler's equations of motion for a rigid body, having no external torque about a fixed point, prove that $T = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}I_3\omega_3^2$ = constant, and $L = I_1\omega_1\hat{i} + I_2\omega_2\hat{j} + I_3\omega_3\hat{k} = \text{constant}$, where the terms have standard meaning.

7. Define generalized coordinates and obtain expression generalized force. The Lorentz force on a particle of mass m and charge q, moving with a velocity $\vec{v}$ in an electric field $\vec{E}$, and a magnetic field $\vec{B}$, is given by

$$\vec{F} = q\left[\vec{E} + (\vec{v}\times\vec{B})\right].$$

If the fields are expressed by the relations: $\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t}$, $\vec{B} = \nabla\times\vec{A}$, ϕ and $\vec{A}$ being the scalar and vector potentials respectively, prove that the Lagrangian for the charged particle is

$$L = \frac{1}{2}mv^2 - q\phi + q\vec{A}\cdot\vec{v}.$$

OR

Explain D'Alembert's principle. Obtain Lagrange's equation of motion for a conservative system using D'Alembert's principle.

8. Derive the differential equation of an orbit in polar coordinates under central force. Show that if the law of central force is defined by $f(r) = -\frac{k}{r^2}, k > 0$, i.e. an inverse square law of attraction, then the path of the particle is a conic.

9. Consider the collision of two particles of mass m_1 and m_2 that stick together on collision. Let m_2 be at rest on the x axis before collision, and let $\vec{v}_{1i} = v_{1i}\hat{i}$ describes the motion of m_1 before the collision.

(a) Describe the motion of $m = m_1 + m_2$ after the collision.

(b) What is the ratio of the kinetic energy after the collision to the initial kinetic energy?

(c) Describe the motion before and after the collision in the centre-of-mass system.

(d) Calculate the loss in kinetic energy in the centre-of-mass system.

OR

What is differential scattering cross section? Show that the differential scattering cross-section for scattering of α-particles by an atomic nucleus is given by

$$\sigma(\Theta) = \frac{1}{4}\left[\frac{ZZ'e^2}{2E}\right]^2 \frac{1}{\sin^4\frac{\Theta}{2}}.$$

10. Write down the Hamiltonian for a simple pendulum and a compound pendulum.
Consider a Lagrangian of the form

$$L = \frac{1}{2}m(\dot{x}^2 - \omega^2 x^2)\, e^{\gamma t},$$

where the particle of mass m moves in one direction. Assume all constants are positive.

(a) Find the canonical momentum and construct the Hamiltonian. Is this Hamiltonian a constant of motion?

(b) Find the equations of motion.