Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination [C]
June, 2018

Level : B. Sc.
Year : II

Course : MCSC 202
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 10

Registration No.:

Date : JUN 19 2018

SECTION "A"
[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The maximum relative error of the number 8.6 is ________________, if the number is correct to given decimal digits.

2. The round off of the number 3.14159 to four significant digits is ________________.

3. The minimum number of iterations in the bisection method to find a solution of the equation $f(x) = 0$ on an interval $[0, 1]$ with a tolerance $\epsilon = 0.01$ is ________________.

4. The order of convergence of the Newton-Raphson method to approximate a solution of the equation $f(x) = 0$ is ________________.

5. If $\Delta y_1 = 1$ and $Ey_0 = 1$, where Δ and E are respectively forward difference and shift operators, then $y_0 =$ ________________.

6. Simpson's 1/3-rule to approximate the value of a definite integral $\int_a^b f(x)dx$ requires the division of the interval $[a, b]$ into a /an ________________ number of sub-intervals of width h.

7. Consider an initial value problem, $\frac{dy}{dx} = xy^2, y(0) = 1$. By Picard's method, the first approximations $y^{(1)} =$ ________________.

8. Gauss-Seidel iterative method converges ________________ as fast as the Jacobi iterative method to solve a system of linear equations $AX = b$.

9. The least square the line $y = a_0 + a_1x$ that fits the given set of data $(x_1, y_1), (x_2, y_2), \dots \dots \dots (x_m, y_m)$ has ________________ normal equations.

10. Newton's forward difference formula gives $\left[\frac{dy}{dx}\right]_{x=x_0} =$ ________________.

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. *(Do not tick the answer)*.

11. The maximum absolute error in the difference of the two number 3.14 and 4.27 correct to given decimal digits is ____________________.
[0, 0.01, 0.005, -1.13]

12. If $y = 4x - 2x^3$, then the maximum absolute error in y at $x = 1$ when error in x is 0.05, is ____________________.
[0.05, 0.5, 0.1, 0.01]

13. One of the real roots of the equation $2x^3 - 3x^2 - 2x + 3 = 0$ is ____________________.
[-1, 0, $\sqrt{2}$, 2]

14. In bisection method, the length of the interval containing the root of the equation $f(x) = 0$ reduces by a factor of ____________________ at each next step.
[1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$]

15. $1 + \Delta =$ ____________________, where the symbols have their usual meanings.
[E^{-1}, ∇, E, δ]

16. If $y = 1 + 2x + 3x^2$, then $\Delta^2 y =$ ____________________, where Δ denotes the forward difference operator.
[6, 3, 2, 1]

17. Bessel's formula for interpolation gives the most accurate result for p in the interval ____________________.
[$-\frac{1}{4} \le p \le \frac{1}{4}$, $\frac{1}{4} \le p \le \frac{3}{4}$, $0 \le p \le \frac{1}{4}$, $-\infty < p < \infty$]

18. The trapezoidal rule approximation of the integration $\int_{x_0}^{x_1} y(x)dx =$ ____________________, where $h = x_1 - x_0$.
[$\frac{1}{2}(y_0 + y_1)$, $\frac{h}{2}(y_0 + y_1)$, $\frac{h}{3}(y_0 + y_1)$, $\frac{1}{3}(y_0 + y_1)$]

19. If $\frac{dy}{dx} = f(x, y)$, then $\frac{d^2y}{dx^2} =$ ____________________.
[$f_x + f_x f_y$, $f_x + f_y$, $f_x + f f_y$, $f_y + f f_x$]

20. A square matrix $A = (a_{ij})$ is upper triangular if ____________________.
[$a_{ij} = 0, j > i$, $a_{ij} = 0, i > j$, $a_{ij} = 0, i = j$, $a_{ij} \neq 0, i = j$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
June, 2018

JUN 1 9 2018

Level : B. Sc.
Year : II
Time : 2 hrs.30 mins.

Course : MCSC 202
Semester : II
F.M. : 50

SECTION "C"
[6 Q. × 7 = 42 marks]

1. Derive a general error formula for a functional relation $z = f(x_1, x_2, \dots, x_n)$ of n variables x_i with respective errors $\Delta x_i, i = 1,2,\dots,n$. [3]
   a) Find the relative error in the function $z = \frac{10x^2}{y}$ at $(x, y) = (1, 1)$ when error in each of x, y is 0.01. [2]
   b) Find the absolute error in the product uv if $u = 56.54 \pm 0.005$ and $v = 4.5 \pm 0.05$. [2]

2. Derive the secant method to approximate a root of an equation $f(x) = 0$ and use this method to approximate a root of the equation $xe^x - 1 = 0$, correct to three decimal digits, starting with initial guesses $x_0 = 0, x_1 = 1$. [4 + 3]

3. Derive a general formula for the numerical integration to approximate the value of a definite integral $\int_{x_0}^{x_n} y(x)dx$. Approximate the integral $\int_0^1 e^{-x^2} dx$ using Simpson's 1/3 rule by dividing the interval $[0, 1]$ into eight equal subintervals.

   OR

   Compute the double integral $\int_0^1 \int_0^1 xy\, dxdy$ using the trapezoidal and Simpson's rules by taking $h = k = 0.5$. Also, find the exact value of the integral and compare the absolute errors in each of these rules. [2+3+2]

4. Derive second order Runge-Kutta method to find the numerical solution of the differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$. Apply this method to tabulate the numerical solution of the differential equation $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$ on the interval $0 \le x \le 0.3$ by taking the step size 0.1. [3+4]

5. Decompose the matrix $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ in the form $A = LU$ where L is lower triangular and U is upper triangular matrix. Hence solve the system of linear equations $AX = b$ where $b = [4\ \ 8\ \ 10]^T$. [3 + 4]

6. Use appropriate interpolation formulas to approximate the values of $f(x)$ at $x = 0.23$, $x = 0.25$ and $x = 0.27$ for following set of tabulated values: [2+3+2]

x	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

7. Derive the normal equations if the straight line $y = a_0 + a_1 x$ is fitted to the data points $(x_i, y_i), i = 1,2 \dots \dots m$ by using least-square curve fitting procedure. Fit the power function $y = ax^b$ using the method of least square for the following data: [4+3]

x	61	26	7	2.6
y	350	400	500	600

SECTION "D"

[4 Q. × 2 = 8 marks]

8. Show that $\delta^2 E = \Delta^2$, where symbols have their usual meanings.

9. Given $f(x) = \frac{1}{x^2}$, find the second order divided difference $[a, b, c]$.

10. From the following table, approximate $\frac{dy}{dx}$ at $x = 0.0$.

x	0.0	0.1	0.2
y	3.01	3.16	3.29

11. Solve the following system of linear equations $2x - y = 7; x + 2y - z = 1; -y + 2z = 1$ using Gauss-Seidel iteration method and perform first four iterations starting with initial guess $(x, y, z) = (3, 3, 3)$.