KATHMANDU UNIVERSITY
End Semester Examination [C]
June/July 2024

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : II
Course : MCSC 202
Semester : II
Exam Roll No. :
Time: 30 mins.
F. M. : 10
Registration No.:
Date : 08 JUL 2024

SECTION "A"
[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. The representation of the number 0.002300 has _____significant digits.

2. Using Newton-Raphson method, if $f(x) = x^2 - 4$ and the initial guess is $x_0 = 2$, the next approximation x_1 is _____.

3. The order of converge of fixed point iteration method to approximate a root of an equation $f(x) = 0$ is _____.

4. For the set of data points $(0,0), (1,1), (2,4)$ of some function $y = y(x)$, the value second order backward difference $\nabla^2 y_2$ = _____.

5. For the interpolation near the middle of a set of finite data points (x_i, y_i) where $x_i = x_0 + ih$, the Bessel's interpolation formula is used if the range of $p = \frac{x-x_0}{h}$ is _____.

6. In numerical integration, the value of the integral of $f(x) = x^2$ from $x = 0$ to $x = 1$ using the trapezoidal rule with $h = 1$ is approximated by_____.

7. By Euler's method to solve $\frac{dy}{dx} = y$ with $y(0) = 1$ and step size $h = 0.1$, the value of y at $x = 0.1$ is _____.

8. The maximum norm $\|A\|_\infty$ of the matrix $A = \begin{pmatrix} -1 & 0 & 2 \\ 0 & -9 & 7 \\ 1 & 4 & 0 \end{pmatrix}$ is _____.

9. In the least square fit to the curve $y = f(x)$ to a given data $(x_i, y_i), i = 1,2,\cdots,m$, the sum of the square of the error $S = \sum_{i=1}^{m}[f(x_i) - y_i]^2$ is _____.

10. The Gauss-Seidel method converges _____than Jacobi method to approximate the solution a system of linear equations.

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (**DO NOT TICK** the answer)

11. In numerical methods, the most commonly used procedures is _____ technique.
[analytic; iterative; reduction; deduction]

12. When the number 1.0001712 is rounded-off to five significant digits, the rounded number will be _____.
[0.0001 1.1712; 1.0001; 1.0002]

13. In Regular-False method, the root is approximated by the x-intercept of the _____line.
[tangent; vertical; chord; horizontal]

14. If interpolation is required near the end of the tabulated data, one should use _____interpolation formula.
[Newton's forward; Newton's backward; Stirling's; Bessel's]

15. Simpson's 1/3 rule is used for _____.
[numerical differentiation; numerical integration;
solving linear equations; interpolating data points]

16. _____method is used to find the root of a nonlinear equations.
[Gauss-Seide; Bisection; Jacobi; LU Decomposition]

17. When applying the Gauss-Seidel method to solve the system of equations $Ax = b$, the coefficient matrix A must be_____to guarantee the convergence.
[diagonally dominant; symmetric; upper triangular; lower triangular]

18. In LU decomposition, a matrix A is factorized into_____.
[two upper triangular matrices; two lower triangular matrices; a lower triangular matrix and an upper triangular matrix; two diagonal matrices]

19. The matrix $A = \begin{pmatrix} -1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{pmatrix}$ is the_____.
[diagonal; singular; upper triangular; lower triangular]

20. The modified Euler's method is _____ order method to solve the ordinary differential equation numerically.
[half; first; second; fourth]

KATHMANDU UNIVERSITY
End Semester Examination [C]
June/July 2024

Level : B.E./B.Sc./B.Tech.
Year : II
Time : 2 hrs. 30mins.

08 JUL 2024

Course : MCSC 202
Semester : II
F. M. : 50

SECTION "C"
[6Q. × 7 = 42 marks]

1. Derive the Newton-Raphson iteration formula to approximate a root of an equation $f(x) = 0$. What is the order of convergence of this method? Use Newton-Raphson method to approximate a root correct to three significant digits of the equation $x^3 - 5x + 3 = 0$ starting with initial guess $x = 2$. Perform four iterations and show all steps clearly. [4+3]

2. Define the terms interpolation and extrapolation. Write the Newton's forward and backward; and Gauss forward and backward interpolation formulas. Use the appropriate one from them to estimate the value of y at $x = 1.5$ and $x = 1.9$ from provided data in the table: [1+2+2+2]

x	1.0	1.2	1.4	1.6	1.8	2.0
y	1.50	2.38	3.42	4.62	5.98	7.50

OR

How would you check a set of data points $(x_i, y_i), i = 1,2, \ldots, n$ is equally or unequally spaced? Name the interpolation formulas that are appropriate for equally and unequally spaced data points. Write the Lagrange's interpolations formula to interpolate a set of points $(x_i, y_i), i = 1,2, \ldots, n$ and discuss the properties of Lagrange's coefficients. Find the Lagrange's interpolation polynomial function $y = f(x)$ from the given data points $(-2, 0), (0, -8), (1, -6)$ and $(2, 0)$ and estimate the value of $f(-1)$. [1+1+2+3]

3. Derive the general formula for numerical definite integral $\int_a^b y(x)\, dx$. How would you get Trapezoidal and Simpson's-1/3 rule as a special case of general formula? Use both Trapezoidal and Simpson's-1/3 rule to estimate the values correct to three decimal places of the definite integral $\int_{-1}^{1} \frac{dx}{2+x}$ by dividing the interval [-1, 1] into eight equal parts and compute the absolute errors by these methods if the exact value is ln 3. [2+1+4]

4. What is LU factorization of a square matrix? How it is used to solve a system of linear equations $Ax = b$? Solve the following system of linear equations $x + y + z = 1; 4x + 3y - z = 6; 3x + 5y + 3z = 4$ by using LU factorization method. [3+4]

5. What is least square curve fitting, and how does it differ from interpolation? Explain the process for performing least square curve fitting. Fit the parabola $y = a + bx + cx^2$ by the method of least square to the data: [3+4]

x	0	1	2	3	4	5
y	5	2	1	2	5	10

6. Mention few numerical methods to solve the first order ordinary differential equation. Discuss the Taylor's series method to solve the differential equation $\frac{dy}{dx} = f(x,y)$ with given the initial condition $y(x_0) = y_0$. Use Taylor's series method to approximate the values of y for $x = 1.1,\ 1.2$ for the equation $\frac{dy}{dx} = xy$ with $y(0) = 1$. [1+3+3]

SECTION "D"

[4Q. × 2 = 8 marks]

7. If $u = x^2y + xy^2$ and errors in x and y are respectively 0.01 and 0.05. Compute the maximum relative error in u at the point $(x,y) = (1,1)$.

8. Prove the relation $\Delta - \nabla \equiv \delta^2$, where Δ, ∇ and δ are the forward, backward and central difference operators.

9. Use Euler's method to get the numerical solution of the differential equation $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$ at the point $x = 0.2$ by taking the step size $h = 0.1$.

10. Estimate the derivative $\frac{dy}{dx}$ at the point $x = 0.1$ of the function $y = f(x)$ using the data points listed in the table:

x	0.1	0.2	0.3	0.4	0.5
y	1.5	2.0	2.5	3.0	3.5