KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Mark Scored:

Level : B.E./B.Sc./B.Tech.
Year : II

Course : MCSC 202
Semester: II

Exam Roll No. :

Time: 30 mins.

F.M. : 10

Registration No.:

Date : 27 JUN 2023

SECTION "A"

[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The value of the difference $\sqrt{2.59}-\sqrt{2.58}$ correct to four significant digits is __________________.

2. The maximum absolute error in the difference $(X_1 - Y_1)$ of the two numbers $X_1 = 3.12$, $Y_1 = 1.34$ given with their respective errors $\Delta x = 0.05$ and $\Delta y = 0.001$ is __________________.

3. The minimum number of iterations in the bisection method to approximate a root of the equation $xe^x - 1 = 0$ on an interval $[0,2]$ with a tolerance $\epsilon = 0.005$ is __________________.

4. The order of convergence of the Newton-Raphson method to approximate a real root of the equation $f(x) = 0$ is __________________.

5. For the set of data points $(0.2, 2.72), (0.4, 2.68), (0.6, 2.88), (0.8, 3.32)$ of some function $y = y(x)$, the value of the second order forward difference $\Delta^2 y_1 =$ __________________.

6. Newton's forward interpolation formula interpolates the most accurate result near the __________________ of set of data points $(x_i, y_i), i = 0,1,\cdots,n$.

7. The column norm $\|A\|_1$ of the matrix $A = \begin{pmatrix} -1 & 2 & 0 \\ 3 & 1 & -2 \\ 1 & -3 & 1 \end{pmatrix}$ is __________________.

8. For the differential equation $\frac{dy}{dx} = x + y$ with initial condition $y(0) = 4$, the value of the second derivative $y''(0) =$ __________________.

9. The Euler's method is __________________ order approximation method to solve the ODE (IVP) $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$ numerically.

10. Two normal equations to fit a straight line $y = a_0 + a_1 x$ to a given data $(x_i, y_i), i = 1,2,\cdots,m$ by the method of least squares are
$a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} y_i = \sum_{i=1}^{m} x_i y_i$ and ________________________.

SECTION "B"

[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. **(DO NOT TICK THE ANSWER)**.

11. If the number X = 3.134 is correct to given decimal places, then the maximum absolute error in the number is __________________.
    [0.5; 0.05; 0.005; 0.0005]

12. The maximum relative error in evaluation of the function $u = 3x^2 - 4$ at $x = 1$ with error of amount 0.01 is__________________.
    [0.6; 0.06; 0.006; -0.06]

13. If a function $f(x) = 0$ has a real root on the interval (a, b), then __________________.
    [$f(a) = 0$; $f(a)f(b) > 0$; $f(a)f(b) < 0$; $f(b) = 0$]

14. The fixed point iteration method $x_{i+1} = \phi(x_i)$ for approximating a root of the equation $f(x) = 0$ converges to an exact root ξ if __________________ for all $x \in I$.
    [$|\phi\ (x)| < 1$; $|\phi'\ (x)| < 1$; $|\phi''(x)| < 1$; $|\phi'\ (x)| = 1$]

15. The backward difference operator ∇ and shift operator E are related by __________________.
    [$\nabla \equiv E - 1$; $\nabla \equiv E$; $\nabla \equiv E^{\frac{1}{2}} - E^{-\frac{1}{2}}$; $\nabla \equiv 1 - E^{-1}$]

16. The Bessel's formula is chosen for the interpolation near the middle of a given set of data points if p takes the values in the range __________________, where $p = \frac{x - x_0}{h}$.
    [$\frac{1}{4} \leq p \leq \frac{3}{4}$; $-\frac{1}{4} \leq p \leq \frac{3}{4}$; $0 \leq p \leq \frac{1}{2}$; $-\frac{1}{4} \leq p \leq \frac{1}{4}$]

17. To use Simpson's 1/3-Rule for numerical integration of definite integral $\int_a^b f(x)dx$, the interval $[a, b]$ must be divided into__________________number of subintervals of width h.
    [odd; even; prime; fractional]

18. The central difference approximation of the first order derivative $y'(x)$ of the function $y = y(x)$ at $x = x_i$ is $y''(x_i) \approx$__________________.
    [$\frac{y_i - y_{i-1}}{h}$; $\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$; $\frac{y_{i+1} - y_{i-1}}{h}$; $\frac{y_{i+1} - y_{i-1}}{2h}$]

19. The least square curve fitting method is particularly useful when the data points __________________.
    [are error free; are evenly spaced; have errors or noise; follow linear trend]

20. The Jacobi iteration method converges __________________ Gauss-Seidel iteration methods to approximate the solution a system of linear equations.
    [twice faster than; twice slower than; equally with; thrice faster than]

27 JUN 2023

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Year : II
Time : 2 hrs. 30 mins.

Course : MCSC 202
Semester: II
F.M. : 50

SECTION "C"

[6Q. × 7 = 42 marks]

1. Discuss briefly the need of study of errors in numerical methods and how error propagates with arithmetic operations. Derive the general formula for the error in the evaluation of a function $u = f(x_1, x_2, \dots, x_n)$ of n arguments $x_i, i = 1, \dots, n$ with error in each x_i being Δx_i. If $u = 3x^3y^2 - 6xy$, find the relative error in u at $(x, y) = (1, 1)$ if the errors in x and y are respectively 0.05 and 0.01. [2+3+2]

2. Derive the Newton-Raphson method to approximate a real root of a non-linear equation $f(x) = 0$ using Taylor's series expansion and interpret it geometrically. Use Newton-Raphson method to obtain a root correct to three decimal places of the equation $x^3 - 5x + 3 = 0$ starting with initial guess $x_0 = 0$. [3+1+3]

3. In the following (x_i, y_i) data there is an error in y −value. Locate and correct the error to re-construct the error free data points (x_i, y_i). Using this improved set of data points, estimate the values of y at $x = 4.3$ by applying the Gauss interpolation formula. [4+3]

x	2.5	3.0	3.5	4.0	4.5	5.0	5.5
y	4.32	4.83	5.27	5.47	6.26	6.79	7.23

OR

Derive the Lagrange's interpolation formula and state the properties of the Lagrange's coefficients. Using this formula find the polynomial which passes through the points $(0, -12), (1, 0), (3, 6), (4, 12)$ and estimate the value of y at $x = 2$. [3+1+3]

4. Derive the normal equations by using the least square procedure for fitting the straight line $y = a_0 + a_1x$ from a given set of data points $(x_i, y_i), i = 1,2, \cdots, m$. Fit the curve $y = ax^b$ by the method of least square for the following data: [2+2+3]

x	2	5	7	11	15	18
y	6.52	8.59	9.50	10.88	11.94	12.61

5. Explain the procedure of Picard's method of successive approximations to solve a differential equation of the form $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$. Use this method to find the series solution of differential equation $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$ and compute the value of $y(0.1)$. [3+3+1]

6. Explain LU decomposition procedure to solve a system of linear equations. Solve the given system of linear equations by using LU decomposition method [2+5]

$$4x + 3y + 2z = 16$$
$$2x + 3y + 4z = 20$$
$$x + 2y + z = 8$$

SECTION "D"

[4Q. × 2 = 8 marks]

7. Use fixed point iteration method to approximate a root of the equation $xe^x = 1$ correct to two decimal places with initial guess $x_0 = 0.5$.

8. The velocity $v(km/min)$ of a train with initial velocity $1\ km/hr$ is given at fixed intervals of time t in minutes as follows:

t	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
v	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0

Estimate approximately the distance covered by a train in 1.0 minute by Simpson's 1/3 rules.

9. Prove the relation $\nabla \equiv \delta E^{-1/2}$, where operators have their usual meanings.

10. Solve by Euler's method, the equation $\frac{dy}{dx} = x + y$ with $y(0) = 0$ to compute $y(0.2)$ and $y(0.4)$ by choosing the step size $h = 0.2$.