KATHMANDU UNIVERSITY
End Semester Examination
February, 2025

Marks Scored:

Level : B.E./B.Sc./B.Tech.		Course : MCSC 202
Year : II		Semester : II
Exam Roll No. :	Time: 30 mins.	F. M. : 10
Registration No.:		Date : 07 FEB 2025

SECTION "A"
[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. If the number X is rounded to N decimal places, then the formula for calculating the maximum absolute error is ____________.

2. The minimum number of iterations required to achieve an accuracy of $\epsilon = 0.001$ to approximate a real root of the equation $f(x) = 0$ in the interval [0, 1] using Bisection method is ____________.

3. The fixed point iteration method $x_{i+1} = \phi(x_i)$ for approximating a root of the equation $f(x) = 0$ converges to an exact root ξ if ____________ for all $x \in I$.

4. If $y_0 = 2, y_1 = 4$ and $y_2 = 2$, then value of second order backward difference $\nabla^2 y_2 =$ ____________.

5. The Stirling's formula is preferred for the interpolation near the middle of a given set of data points if p takes the value in the range ____________, where $p = \frac{x-x_0}{h}$.

6. The row norm $\|A\|_\infty$ of the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 3 & 1 & 2 \\ -2 & 3 & -1 \end{pmatrix}$ is ____________.

7. The finite difference approximation of the second order derivative $y''(x)$ of the function $y = y(x)$ at $x = x_i$ is $y''(x_i) \approx$ ____________ if the length of each subinterval is $h = x_{i+1} - x_i$.

8. For the differential equation $\frac{dy}{dx} = x^2 + y^2$ with initial condition $y(1) = 1$, the value of the second order derivative $y''(0) =$ ____________.

9. Two normal equations to fit the straight line $y = a + bx$ to a given data $(x_i, y_i), i = 1,2,\cdots,m$ by the method of least squares are ____________ and ____________.

10. The Jacobi method converges ____________ than Gauss-Seidel method to approximate the solution a system of linear equations.

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (***DO NOT TICK THE ANSWER***)

11. The relative error of the number 8.6 correct to given digits is ______________.
[0.05; 0.058; 0.005; 0.0058]

12. When the number 2.375 is rounded-off to two decimal places, the rounded number will be ______________.
[2.4; 2.38; 2.37; 2.36]

13. The Generalized Newton's method to approximate a root of an equation $f(x) = 0$ with multiplicity p reduces to the Newton-Raphson method if $p =$ ______________.
[0; 1; 2; 3]

14. The order of convergence of the secant method to approximate a real root of the equation $f(x) = 0$ is ______________.
[$\frac{1+\sqrt{5}}{2}$; $\frac{1-\sqrt{5}}{2}$; 1.5; 2]
______________.

15. The forward difference operator Δ and shift operator E are related by
[$E \equiv 1 + \Delta$; $E \equiv 1 - \Delta$; $E \equiv \Delta - 1$; $E \equiv \Delta$]

16. Newton's forward interpolation formula interpolates the most accurate result near the ______________ of set of data points $(x_i, y_i), i = 0,1, \cdots, n$.
[end; center; anywhere; beginning]

17. The sum of the Lagrange coefficients $\sum_{i=0}^{n} l_i(x) =$ ______________ in Lagrange interpolation formula for a set of data point $(x_i, y_i), i = 0,1, \ldots, n$.
[0; 1; -1; ∞]

18. The Jacobi and Gauss-Seidel iterative methods guarantee convergence for solving the system of linear equations $AX = b$ if the condition ______________ $i = 1,2 \ldots n$ is met.
[$\sum_{j=1, j\neq i}^{n} |a_{ij}| < 1$; $\sum_{j=1, j\neq i}^{n} |a_{ij}| > |a_{ii}|$; $\sum_{j=1, j\neq i}^{n} |a_{ij}| < |a_{ii}|$; $\sum_{j=1, j\neq i}^{n} |a_{ij}| < 0$]

19. The interval $[a, b]$ in Simpson's 1/3-Rule for numerical integration of definite integral $\int_a^b f(x)dx$ should be divided into______________number of subintervals of width h.
[odd; any; prime; even]

20. The Euler's method is ______________order approximation method to solve the ODE (IVP) $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ numerically.
[first; second; third; fourth]

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Time : 2 hrs. 30mins.	F. M. : 50

SECTION "C"
[6Q. × 7 = 42 marks]

1. Derive Newton-Raphson method to approximate a real root of the equation $f(x) = 0$ and interpret the method geometrically. Use Newton-Raphson method to find a root of the equation $x^3 + x^2 + x - 7 = 0$ correct to three significant digits with the initial guess $x_0 = 1$. [4+3]

2. Write the Gauss forward and Gauss backward interpolation formulas. How would you get Stirling's formula from Gauss interpolation formulas? Use Stirling's formula to estimate the value of y and its derivative $\frac{dy}{dx}$ at $x = 1.18$ from the following table: [2+1+2+2]

x	1.00	1.05	1.10	1.15	1.20	1.25	2.2
y	2.72	2.86	3.00	3.16	3.32	3.49	3.67

OR

Derive Lagrange interpolation formula for given set of data point $(x_i, y_i), i = 0, 1, 2, \ldots, n$ and discuss the properties of Lagrange's coefficients. How Lagrange interpolation formula is used for inverse interpolation? Use Lagrange's interpolation formula from the given data points $(-2, 0), (0, -8), (1, -6)$ and $(2, 0)$ of function $y = f(x)$ to get the interpolating polynomial and use it to estimate the value of $f(1.5)$. [3+1+3]

3. Derive the general formula for numerical definite integral. Estimate the value of the double integral $\int_0^1 \int_0^1 e^{x+y} dx\, dy$ using the Trapezoidal and Simpson's rules taking $h = k = 0.5$. Also, find the absolute error by comparing the results with the exact value. [2+2+2+1]

4. Explain the LU decomposition procedure to solve a system of linear equations $AX = b$. Solve the system of equations: $2x + y + z = 4,\ x - 2y + 4z = 3,\ x + 3y - 2z = 2$ using LU decomposition method. [2+5]

5. Derive the normal equations by the method of the least square to fit the line $y = a_0 + a_1 x$ from a given set of data points $(x_i, y_i), i = 1, 2, \cdots, m$. Using the given data, fit the curve $y = ax^b$ by the method of least square and also estimate the value of y at $x = 10$. [3+3+1]

x	2	5	8	11	15	18
y	1.4	5.6	11.3	18.2	29.0	38.2

P.T.O.

6. Derive modified Euler's method for estimating the solution of differential equation $\frac{dy}{dx} = f(x, y)$ given the initial condition $y(x_0) = y_0$. Use modified Euler's method to approximate the solution of differential equation $\frac{dy}{dx} = x + y$ with $y(0) = 1$ on the interval $0 < x \leq 1$ by taking the step size 0.25. [3+4]

SECTION "D"

[4Q. × 2 = 8 marks]

7. If $u = \frac{1}{8}xy^3$, find the percentage error in u when $x = 3.14 \pm 0.0016$ and $y = 4.5 \pm 0.05$.

8. Apply fixed point iteration method to approximate a root correct to two decimal places of the equation $e^x = 4x$, starting with initial guess $x_0 = 0$ (Perform only four iteration steps).

9. Prove the relation $\mu^2 = 1 + \frac{1}{4}\delta^2$, where μ and δ respectively denote the mean and central difference operators.

10. Use Picard's successive approximation method to get the series solution of the differential equation $\frac{dy}{dx} = -y$ with $y(0) = 1$.