KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2018

Marks Scored:

Level : B.Sc.
Year : II

Course : MCSC 202
Semester: II

Exam Roll No.: Time: 30 mins. F.M : 10

Registration No.: Date MAR 0 4 2018

SECTION "A"

[10 Q.×0.5=5 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The number 0.023657 when rounded-off to four significant digits is ....

2. If the numbers X and Y have errors of amount -0.01 and 0.01 respectively, then maximum absolute error in the sum $X+Y$ is ....

3. The minimum number of iterations in the bisection method to find a root of the equation $f(x)=0$ lying on an interval $[0,1]$ with a tolerance $\epsilon = 0.005$ is ....

4. The first approximated value of the root of the equation $x^2-4=0$ using the method of false position, if the initial guesses are 3 and 4, is ....

5. The symbolic relation of the averaging operator μ with the shift operator E is $\mu \equiv$.................... .

6. If $\Delta y_r = 3$ and $Ey_r = 2$, then $y_r =$...................., where Δ and E denote the forward difference and the shift operators respectively.

7. If $y = 3x^2 - 2x + 1$, then $\Delta^2 y =$...................., where Δ denotes the forward difference operator.

8. Simpson's 1/3 rule to approximate the value of a definite integral $\int_a^b f(x)dx$ requires the division of the interval $[a, b]$ into a /an .... number of sub-intervals.

9. The Euler's method approximates solution $y(0.5)$ of the initial value problem $y' = x + cosy, y(0) = 1$ with $h = 0.5$ is ....

10. The Gauss-Seidel method converges approximately .... faster than the Jacobi method.

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (*Do not tick the answer*).

11. If the number $Z = 12.025$ is given correct to three decimal digits, then the maximum absolute error in the number is ....
[0.05; 0.005; 0.0005; 0.00005]

12. If $y = 3x^7 - 6x$, then the relative error in y at $x = 1$, when error in x is 0.01, is ....
[0.05; 0.25; 0.5; 0.01]

13. The fixed point iteration $x_{n+1} = \phi(x_n), n = 0,1,2, \ldots$ converges to the root $x = \xi$ of the equation $f(x) = 0$, if .... for all x.
[$|\phi'(x)| = 1$; $|\phi'(x)| < 1$; $|\phi'(x)| > 1$; $|\phi'(x)| = 0$]

14. The maximum order of convergence of the Newton-Raphson method to approximate a root of a equation $f(x) = 0$ is ....
[0; 1; 2; 3]

15. For the most accurate result to interpolate near the middle of the given set of equally spaced data points, the best choice is .... interpolation formula when $\frac{1}{4} \leq p \leq 3/4$, where $p = \frac{x - x_0}{h}$.
[Newton's backward; Newton's forward; Stirling's; Bessel's]

16. For the given set of points $(x_i, y_i), x_i = x_0 + ih, i = 0,1,2, \ldots \ldots$, the relation of the n^{th} order divided difference $[x_0, x_1, \ldots, x_n]$ with the n^{th} order forward difference is ....
[$\frac{\Delta^n y_0}{h^n}$; $\frac{\Delta^n y_0}{n!}$; $\frac{\Delta^n y_0}{h^n n!}$; $\frac{\Delta^n y_0}{h^n n}$]

17. The highest degree of polynomial integrand, for which Trapezoidal rule of numerical integration is exact, is ....
[first; second; third; fourth]

18. The second order finite difference approximation of the derivative of the function $f'(x_i) \approx$............................ .
[$\frac{f(x_i) - f(x_{i-1})}{h}$; $\frac{f(x_{i+1}) - f(x_i)}{h}$; $\frac{f(x_{i+1}) - f(x_{i-1})}{h}$; $\frac{f(x_{i+1}) - f(x_{i-1})}{2h}$]

19. The Jacobi and Gauss-Seidel iteration methods converge to the exact solutions of the system of linear equations $AX = b$, where $A = (a_{ij})_{1 \leq i,j \leq n}$ if .... for all $i = 1,2, \ldots, n$.
[$\sum_{j=1, j \neq i}^{n} |a_{ij}| > |a_{ii}|$; $\sum_{j=1, j \neq i}^{n} |a_{ij}| = |a_{ii}|$; $\sum_{j=1, j \neq i}^{n} |a_{ij}| = 0$; $\sum_{j=1, j \neq i}^{n} |a_{ij}| < |a_{ii}|$]

20. For a given $(x_i, y_i), i = 1,2, \ldots, m$ data points, best fitting data to a curve $y = f(x)$ by the method of least squares requires the minimization of ....
[$\sum_{i=1}^{m} [y_i - f(x_i)]$; $\sum_{i=1}^{m} [|y_i - f(x_i)|]$; $\sum_{i=1}^{m} [y_i - f(x_i)]^2$; $\sum_{i=1}^{m} [y_i - \bar{y}]^2$]

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MAR 0 4 2018

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MCSC 202
Semester: II
F.M. : 50

SECTION "C"
[6 Q.×7=42 marks]

1. Explain the Bisection method to approximate a root $\xi \in [a, b]$ of the equation $f(x) = 0$. Also derive the formula for the number of iteration steps n required to make the length of the sub-interval in the n^{th} step is less than a small positive number ϵ.
Approximate a root on the interval $[0, 1]$, correct to two decimal places of the equation $x^2 + x - \cos x = 0$, using Bisection method. [3+1+3]

2. Obtain the general numerical integration formula to approximate the definite integral $\int_a^b f(x)dx$, and hence derive the Simpson's-1/3 rule for numerical integration. Use Simpson's-1/3 rule to approximate the integral $\int_0^1 \frac{dx}{1+x^2}$ with 8 subintervals. [3+2+2]

3. Derive the second order Runge-Kutta method to approximate the solution of the initial value problem $\frac{dy}{dx} = f(x,y)$, with initial condition $y(x_0) = y_0$. Use this method to approximate the solution of $\frac{dy}{dx} = xy + y^2$ with $y(0) = 1$ on the interval $0 < x \leq 0.6$ by taking $h = 0.2$. [3+4]

4. What is an interpolation? State the Gauss' forward and Gauss' backward interpolation formulas and hence establish the Stirling's formula. Use Stirling's formula to estimate the value of $f(0.25)$ for the given set of points

x	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

[1+3+3]

OR

Derive Newton's divided-difference interpolation formula and hence deduce the Newton's forward difference interpolation formula as a particular case.
For the given set of points $(0, 2), (1,3), (2,12)$ and $(15,3587)$ satisfying the function $y = f(x)$, compute $f(4)$ using Newton's divided difference interpolation formula. [3+1+3]

5. Find the second order finite difference approximations of the first and the second derivatives of a function $y = f(x)$ at $x = x_i$ using Taylor's series.
Solve the boundary value problem defined by $y'' - y = 0$, with $y(0) = 0$ and $y(1) = 0$ on the interval $0 < x < 1$ taking $h = 0.25$ by using finite difference method. [2+5]

6. Derive the normal equations to fit the straight line $y = a_0 + a_1 x$ for the given set of points $(x_i, y_i), i = 1,2, \dots, m$ by using least-square curve fitting procedure.
Use the method of least squares to fit a function of the form $y = ax^b$ for the following set of points

x	61	26	7	2.6
y	350	400	500	600

Also, estimate y for $x = 15$. [3+4]

SECTION "D"

[4 Q.×2=8 marks]

7. If $z = \frac{1}{8}xy^3$, find the maximum relative error in z at $x = 3.14, y = 4.5$ when absolute errors in x and y are 0.0016 and 0.05 respectively.

8. The distance ($x\ cm$) traversed by a particle at different times ($t\ seconds$) are given below in the table:

t	0.0	0.1	0.2	0.3	0.4
x	3.01	3.16	3.29	3.36	3.40

Find the velocity of the particle at $t = 0.2$ seconds.

9. Solve the system $6x + y + z = 20; x + 4y - z = 6; x - y + 5z = 7$ using Gauss-Seidel method to get first three approximations by assuming the initial guesses (x, y, z) =(0, 0, 0).

10. Decompose the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix}$ so that $A = LU$, where L is unit lower triangular matrix and U is upper triangular matrix.