KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

Marks Scored:

Level : B.E./B.Sc./B.Tech. | Course : MCSC 202
Year : II | Semester : II
Exam Roll No.: | Time: 30 mins. | F. M. : 10
Registration No.: | Date : 28 NOV 2023

SECTION "A"

[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. The absolute error in the quotient a/b is given by the formula _____.
2. Geometrically, the Newton-Raphson method approximates the value by means of _____.
3. The relation between the mean operator (μ) and the central difference operator (δ) is defined by the relation _____.
4. Bessel's formula gives the most accurate result in the interval _____.
5. Stirling's formula for $\left.\frac{dy}{dx}\right|_{x=x_0}$ = _____.
6. The maximum norm for the vector $\vec{X} = [2{,}8, -32{,}20, -35{,}25]$ is _____.
7. The convergence criteria of Gauss-Seidel method for the system of n −linear equations satisfies the conditions _____.
8. For given differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}, y(0) = 0$, the first approximation using Picard's method is _____.
9. Let $(x_i, y_i), i = 1,2,3, \ldots, n$ be the set of observed data points and $y = f(x)$ be fitted curve to this data. Then the method of least squares minimizes _____.
10. Consider initial value problem $\frac{dy}{dx} = 1 + y^2, y(0) = 0$ & $h = 0.4$. According to Runge-Kutta second order method $y(0.4)$ = _____.

SECTION "B"

[10 Q.× 0.5 = 5 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **Do not tick** the answers

11. If the error at two consecutive steps satisfies the asymptotic relation $|\epsilon_5| = 3|\epsilon_4|^2$, then the order of convergence will be _____.
[2: 3: 4: 5]

12. If $X = 0.51$ is correct to 2 decimal places, then it's relative accuracy is_____.
[0.0096; 0.0097; 0.0098; 0.0099]

13. The iteration method for the equation $x = \phi(x)$ converges at $x = x_i$ if $|\phi'(x_i)| \leq k$ such that _____.
[$k < 1$; $k \leq 1$; $k > 1$; $k \geq 1$]

14. The relation between backward and shift operator is _____.
[$\nabla = E - 1$; $\nabla = 1 + E$; $\nabla = E^{-1} - 1$; $\nabla = 1 - E^{-1}$]

15. Third order divided difference for $(n + 1)$ data points $(x_0, y_0), (x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$ is denoted by _____.
[$[x_0, x_1, x_2]$; $[x_0, x_1, x_2, x_3]$; $[x, x_0, x_1]$; $[x, x_0, x_1, x_2]$]

16. The norm of a matrix A is defined by the formula $\max_i \Sigma_j |a_{ij}|$ is denoted by _____.
[$\|A\|_i$; $\|A\|_\infty$; $\|A\|_2$; $\|A\|_e$]

17. The forward difference approximation for $y'(x)$ is represented by _____.
[$\frac{y(x+h)-y(x)}{h}$; $\frac{y(x+h)-y(x)}{2h}$; $\frac{y(x+h)-y(x-h)}{2h}$; $\frac{y(x)-y(x-h)}{h}$]

18. The number 0.000145 have _____ significant digits.
[6; 5; 4; 3]

19. For a given matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, $\|A\|_1 =$_____.
[18; 16.88; 24; 20]

20. Lagrange interpolation coefficient satisfies the property _____.
[$l_1(x_1) = 0$; $l_1(x_2) = 1$; $l_2(x_1) = 1$; $l_1(x_1) = 1$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
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28 NOV 2023

Level : B.E./B.Sc./B.Tech.
Year : II
Time : 2 hrs. 30 mins.

Course : MCSC 202
Semester : II
F. M. : 50

SECTION "C"
[6 Q. × 7 = 42 marks]

1. Derive a formula to find the root of an equation $f(x) = 0$ by secant method. Use this method to find a root of the equation $x^5 + x^3 + 3 = 0$ correct to 4 decimal places using initial approximations -2 and -1.

2. Derive Newton's divided difference interpolation formula. From the following table of values, estimate $f(2.4)$, correct to 4 decimal places and compute the percentage error if exact value is 0.380211:

x	1	1.5	2	3	3.5
$f(x)$	0	0.17609	0.30103	0.47712	0.54407

3. The weight, W, of a metallic object decreases over time when exposed to a caustic environment according to the exponential law $W = ae^{-t/\tau}$, where t is the exposure time and τ is known as the decay rate constant. Data for a group of objects made from the same material is given in the following table:

Exposure time (days)	5	10	15	20	25	30	35	40
Weight (grams)	92.7	58.3	59.5	41.7	45.6	31.8	38.3	19.9

Find the decay rate constant, τ, for this material and estimate the weight at 60 days.

4. Discuss LU decomposition method to solve a system of linear equation $A\vec{X} = \vec{B}$. Solve the following system using LU-decomposition:

$$7x + 3y - z = 3$$
$$3x + 8y + z = -4$$
$$-x + y + 4z = 2$$

5. Solve the following integral using Simpson's 1/3 rule and estimate the error: $\int_0^2 \int_{-2}^3 e^{x+y}\, dydx$.

6. The following initial value problem has been proposed as a model for a genetic switch $\frac{dg}{dt} = s + 2.43\frac{g^2}{1+g^2}, g(0) = 0$, where s denotes the concentration of chemical and g denotes concentration of protein product. Estimate $g(10)$ using RK4 method for $s = 0.2$. Approximate the value on $n = 5$ subintervals.

OR

Solve using finite difference method using step size $h = 0.25$ the boundary value problem: $xy'' - (x+5)y' + 4y = x, y(1) = -1, y(2) = 1$

SECTION "D"

[4 Q. × 2 = 8 marks]

7. Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find it's absolute and relative errors.

8. Find the root of $2x = \cos x + 3$ correct to 2 decimal places using fixed point iteration method. The initial approximation is $x_0 = \pi/2$.

9. Find a quadratic polynomial passing through the data: $(0,7), (5,17), (10,77)$.

10. Find dy/dx at $x = 1.6$ for the following data values:

x	1	1.2	1.4	1.6
y	2.7183	3.3201	4.0552	4.9530