SEP 07 2017

Marks scored:

KATHMANDU UNIVERSITY
End Semester Examination
August/September, 2017

Level: B.E./B.Sc./B. Tech. — Course : MCSC 202
Year : II — Semester: II

Exam Roll No. : — Time: 30 mins. — F. M. : 10
Registration No.: — Date :

SECTION "A"
[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. For the matrix $A=\begin{bmatrix}-5 & 2 & 6\\ 3 & -1 & -4\\ 5 & 6 & 3\end{bmatrix}$, the column norm $\|A\|_1 =$ ______________.

2. If an approximate value of π is given by 3.1428571 and its true value is 3.1415926. Then the relative error is given by ______________________.

3. Given an initial value problem $y' = x + y^3$, $y(0)=1$, the value of $y(0.1)$, correct to three decimal places, using the Runge-Kutta second order formula is ______________________when x-step size h = 0.1.

4. $\Delta x^2 =$ ______________________, where Δ is the forward difference operator, and h is the x - step size.

5. The order of convergence of the fixed point iteration method for the solution of an equation $f(x)=0$ is ______________________.

6. The volume of a solid formed is given by $V = \pi \int_0^{\pi} y^2 dx$. The value of y^2 corresponding to values of x are tabulated below:

x	0.00	0.25	0.50	0.75	1.00
y	1.0000	0.9793	0.9195	0.8261	0.7081

Then the value of V using the Simpson's $\frac{1}{3}$-rule is ______________________.

7. The Newton Raphson method to find $\sqrt{N}$, $N>0$ is given by $x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$, $n = 0,1,2,\ldots$.

The value of x_2 of $\sqrt{12}$, correct to four decimal places, given that $x_0 = 3.5$ is ______________________.

8. The divided difference $[x_0, x_0]$ of the tabulated values $(x_i, y_i), i = 0,1,2,\ldots$ of the function $y = f(x)$ is ______________________.

9. The first approximate series solution to the initial-value problem $y' = 1 + y^2, y(0) = 0$ using Picard's iteration formula is $y_1 =$ ____________________.

10. If a number X is rounded to N decimal places, then the maximum absolute error is given by $\Delta X =$ ____________________.

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (*Do not tick th answer*).

11. The order of convergence of the secant method to find the root of $f(x) = 0$ is ____________________.
[1, 1.62, 2, 3]

12. The Gauss-Seidel method converges ____________________ times faster than the Jacobi method.
[two, three, four, five]

13. The Gauss backward interpolation formula is used to interpolate the value of the function for the value of p such that ____________________, where the symbol has its usual meaning.
[$0 < p < 1$, $-1 < p < 0$, $-1 < p < 1$, $-\frac{1}{2} < p < \frac{1}{2}$]

14. The first derivative of $f(x)$ at $x = 0.3$ from the following table

x	0.1	0.2	0.3
y	1.10517	1.22140	1.34986

is ____________________.
[0.134575, 0.269561, 1.22345, 2.32862]

15. Suppose the approximate value of $y(x)$ at $x = x_i$ is y_i. Then the difference scheme for y_i' defined by $y_i' = \frac{y_{i+1} - y_i}{h}$ is known as ____________________ difference approximation.
[forward, backward, central, two-point]

16. The rounding-off the number 0.032552 to three significant digit is ____________________.
[0.032, 0.033, 0.0325, 0.0326]

17. The least square method consists in ____________________ the sum of the squares of the errors.
[maximizing, minimizing, optimizing, squaring]

18. The minimum number of iterations required for converging to a root in the interval (0, 1) for a give tolerance, $\in = 10^{-2}$ in the Bisection method is ____________________.

[5, 6, 7, 8]

19. Let $y(x)$ be a polynomial of nth degree with leading coefficient A_0. Then $\Delta^n y(x) =$ ________________, where h is the x-step size and Δ is the forward difference operator.

[0, A_0, $A_0 n!$, $A_0 n! h^n$]

20. The Simpson's $\frac{1}{3}$ rule for numerical integration $\int_a^b y dx$ is used when the number of sub-intervals of the interval $[a,b]$ is a multiple of ________________.

[2, 3, 4, 5]

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F. M. : 50

SECTION "C"
[6 Q. × 7 = 42 marks]

1. Discuss LU factorization method to find the solution (x, y, z) of the system of linear equations. Use LU factorization method to find the solution, correct to four decimal places, of the following system of linear equations: [3 + 4]

$$4x+y+z=2, \quad x+5y+2z=-6, \quad x+2y+3z=-4$$

2. Derive the Newton's forward difference interpolation formula. The following values of the function $f(x)=\sin x+\cos x$ are given

x	10^0	20^0	30^0
$f(x)$	1.1585	1.2817	1.3660

Construct the quadratic interpolating polynomial that fits the data. Hence find $f\left(\frac{\pi}{2}\right)$, and compare with the exact value. [3 + 3 + 1]

3. Show that the Newton-Raphson method for a root of the equation $f(x) = 0$ is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Also, show that this method converges quadratically in the neighborhood of the root of $f(x) = 0$. Find a root of the equation $x \sin x + \cos x = 0$, correct to four decimal places, using Newton-Raphson method with initial guess $x_0 = \pi$ and tolerance is 10^{-4}. [2 + 2 + 3]

OR

Describe the fixed point iteration method for the solution of the two system of equations $f(x, y) = 0$ and $g(x, y) = 0$. Also, Solve the system of nonlinear equations $x = -0.1x^2 + 0.1y + 0.5$ and $y = 0.1x + 0.1y^3 + 0.1$ correct to 0.001 with initial approximations $x_0 = 1$ and $y_0 = 1$. [3 + 4]

4. Derive the Euler method for the solution of an initial value problem $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$. Use the Euler method to find the value of $y(0.1)$, correct to four decimal places, of the initial value problem $\frac{dy}{dx} = x^2 + y, y(0) = 1$ with step size $h = 0.02$. [3 + 4]

5. Derive the numerical differentiation formula for $\left[\frac{dy}{dx}\right]_{x=x_0}$ of a given set of $(n+1)$ tabulated values $(x_i, y_i), i = 0,1,\ldots,n$. Find the first derivative of $f(x)$ at $x = 0.1$ from the following table: [4+3]

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

6. What do you mean by a curve fitting? Describe least square curve fitting of a straight line $y = a_0 + a_1 x$. Find the values of a_0 and a_1 so that $y = a_0 + a_1 x$ fits the data given in the table: [1+3+3]

x	0	1	2	3	4
y	1.0	2.9	4.8	6.7	8.6

SECTION "D"

[4 Q. × 2 = 8 marks]

7. Prove that the relative error of a product of two non-zero numbers does not exceed the sum of the relative errors of the given numbers.

8. Show that $\Delta \equiv E - 1$, where the symbols have their usual meanings.

9. Derive the central difference approximations for $y'(x)$.

10. Evaluate $\int_0^1 \frac{1}{1+x} dx$ using Trapezoidal rule when step size $h = \frac{1}{8}$.