KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

02 JUN 2019

Level : B.E./B.Sc.
Year : II

Course : MCSC 201
Semester: I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date :

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) with the most appropriate answer(s):

1. If p is a statement $2 \geq 3$, then negation of p is ______________________.

2. Let S be a non-empty set together with a binary operation $*$, and e is an identity element under the operation $*$ in S, and $x, y \in S$ are inverse to each other under the operation $*$. Then $x * y = y * x =$ ______________________.

3. If $U = \{a, b, c, d, e, f\}$ and $A = \{a, c, e\}$, then the finite sequence f_A is ______________, where f_A is characteristic function on A.

4. The range of the characteristics function defined on subset A of universal set U is ______________________.

5. If $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (b, a), (b, b), (c, d), (d, c), (c, c), (d, d)\}$ is a relation then the quotient set A/R is ______________________.

6. If M_R be a matrix representation of symmetric relation R on a set A, and R^{-1} be an inverse relation of R, then $M_{R^{-1}} =$ ______________________.

7. A path in a graph G is called a ______________________ if it includes every edge exactly ones.

8. The value of the ceiling function $\lceil x \rceil$ at $x = 2.7$ is ______________________.

9. A group $(G, *)$ is said to be abelian if ______________________ for each a, b in G.

10. If the function $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5x + 1$, then $f^{-1}(6) =$ ______________________.

SECTION "B"

[10Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from the given ones.

11. If 31 pigeons are assigned to 5 pigeonholes then one of the pigeonholes must contain at least ______________________ pigeons.
[3; 4; 5; 6]

12. The number of distinguishable permutations of letters in "MANANDHAR" is ________.
[40320; 30240; 40330; 40230]

13. If $A = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$, then $2B - A =$ ________.

[$\begin{bmatrix} 0 & 3 \\ -5 & 8 \end{bmatrix}$; $\begin{bmatrix} 0 & -3 \\ 5 & 8 \end{bmatrix}$; $\begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$; $\begin{bmatrix} 0 & 3 \\ -5 & -8 \end{bmatrix}$]

14. The probability that when two dice are rolled, the sum of the numbers on the two dice being 10, is ________.
[1/2 ; 1/4; 1/9; 1/12]

15. Let L be a lattice. Then, for each a, b and c in L, $a \leq c$ and $b \leq c$ if and only if ________, where the symbols have their usual meanings.
[$a > b \vee c$; $a \geq b \vee c$; $a \vee b < b$; $a \vee b \leq c$]

16. The matrix $M_R = [m_{ij}]$ is a matrix representation of a reflective relation R, then $m_{ij} =$ ________, for $i = j$.
[$m_{ij} = 1$; $m_{ij} = -1$; $m_{ij} = 0$; $m_{ij} = \infty$]

17. If f is the mod- 5 function, then f(1217530) = ________.
[0; 1; 2; 3]

18. In the permutation group S_3, inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$ is ________.

[$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$; $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$; $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$; $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$]

19. If A = {a, b, c, d}, then n(P(A)) ________, where P(A) is the power set of A.
[8; 16; 24; 32]

20. If the mappings $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = x$, $g(x) = x + 3$, then $(gof^{-1})(-1) =$ ________.
[2; 3; -2; -3]

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Level : B.E./B.Sc. Course : MCSC 201
Year : II Semester : I
Time : 2 hrs. 30 mins. F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Define prime number and GCD of two positive integers. Let a, b, c be integer, $a \mid b$ or $a \mid c$ then prove that $a \mid bc$. Also, use Euclidian algorithm to find GCD of 273 and 98 and express it as a combination of given numbers. [2 + 2 + 3]

2. Define matrix relation and an equivalence relation of a relation R on a set A. Let R be an equivalent relation on a set A then for each $a, b \in A$, prove that $a\,R\,b$ if and only if $R(a) = R(b)$. Find M_{R^2}, if R be the relation on set A whose matrix is $M_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. [2 + 3 + 2]

OR

Define a graph, sub-graph and Euler circuit. If a graph G has a vertex of odd degree, then prove that there can be no Euler circuit in G. [1 + 1 + 1 + 4]

3. Define semi-group, group and monoid. Let f be a homomorphism from a semi-group $(S,*)$ to $(T,*')$. If S' is a submonoid of S, then show that $f(S') = \{t \in T: t = f(s),$ for some s in $S'\}$, the image of S' under f is a submonoid of T. [1 + 1 + 1 + 4]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p = (1 \quad 2 \quad 4 \quad 3)$ and $q = (5 \quad 6)$ be permutations of A. Compute p^{-1}, p^2 and $p \circ q$.

5. Define the recurrence relation. Find an explicit formula for the sequence defined by $a_n = 2a_{n-1} + 1$ with initial condition $a_1 = 7$.

6. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then compute $A \vee B$, $A \wedge B$ and $A \oplus B$, where the symbols have their usual meanings.

7. If $(A, \leq)$ and $(B, \leq)$ posets, then show that $(A \times B, \leq)$ is poset with partial order $\leq$ define by $(a, b) \leq (a', b')$ if $a \leq a'$ in A and $b \leq b'$ in B.

OR

Define the characteristic function and prove that $f_{A \oplus B} = f_A + f_B - 2\, f_A\, f_B$, where the symbols have their usual meaning.

8. Use Mathematical Induction to prove that $1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all non-negative integer n.

9. Verify that the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 5x + 7$ satisfy the property $f^{-1} o\ f = I_{\mathbb{R}}$ an identity on $\mathbb{R}$.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Let A and B be subsets of U, then prove that $\overline{A \cup B} = \bar{A} \cap \bar{B}$.

11. If (G, *) is a group where G is the set of all non -zero real numbers and $a * b = \frac{ab}{2}$ for all a, b in G. Find the identity element of G.

12. Show that the statement $(p \wedge q) \Rightarrow q$ is a tautology.

13. Let A = $\{a, b, c\}$ and R be a relation on A, whose matrix relation is

$$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$ Find the $M_{R^{-1}}$ and $M_{\bar{R}}$.

14. Define the maximal element of a poset. What are the least and greatest elements of M if S = $\{a, b, c\}$ and poset M =$(P(S), \subseteq)$.