KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

Marks Scored:

Level : B. E./B. Sc.
Year : II

Course : MCSC 201
Semester : I

Exam Roll No. :

Time : 30 mins.

F. M. : 20

Registration No. :

Date APR 11 2017

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate word(s) or symbol(s):

1. Two statements p and q are equivalent if "p $\Leftrightarrow$ q" is ........ ............ .

2. The expression $0^*(0 \vee 1)^*$ over a set A is a(an) .................. expression.

3. The range of the characteristics function defined on subset A of universal set U is ........

4. A pictorial representation of a relation on a finite set is .......... of the relation.

5. If A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3), (3, 3), (4, 4)} is a relation then the quotient set A/R is ........................

6. The recursive formula for the sequence 2, 5, 8, 11, 14, 17, ... is ....................

7. A graph with each vertex having the same degree is .............

8. $\lfloor -27.33 \rfloor$ = ..................

9. A semi-group (S, $*$) with identity element is ...................

10. If the function $g : \Re \to \Re$ be defined by $g(x) = x^2 + 1$, then $f^{-1}(10)$ = ...............

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by choosing the most appropriate answer from the given ones.

11. If 37 pigeons are assigned to 7 pigeonholes then one of the pigeonholes must contain at least ........... pigeons.
[ 3 ; 4 ; 5 ; 6 ]

12. The number of distinguishable permutations of letters in "MATHEMATICS" is .................... .
[ 4989200 ; 4989400 ; 4989600 ; 4989800 ]

13. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, then $A \otimes B =$ ...........

[ $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ; $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ; $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ ; $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ]

14. If a graph G with 5 vertices has a Hamiltonian circuit then G must has at least ............ edges.
[ 2 ; 3 ; 4; 5 ]

15. Let ***L*** be a lattice. Then, for each a, b in ***L***, $a \vee b = b$ if and only if ..............
[ $a > b$ ; $a \geq b$ ; $a < b$ ; $a \leq b$ ]

16. The matrix $M_R = [m_{ij}]$ is an asymmetric relation R has the property that if $m_{ij} = 1$, then ..............
[ $m_{ji} = 1$ ; $m_{ji} = -1$ ; $m_{ji} = 0$ ; $m_{ij} = 0$ ]

17. A rooted tree $(T, v_0)$ on a set A is ...........
[ reflexive ; symmetric ; asymmetric ; antisymmetric ]

18. The group $S_4$ is a group of order .............
[ 8 ; 12 ; 16 ; 24 ]

19. If $f_2(n) = 2^n$, then $f_2(10) =$ .............
[ 1022 ; 1024 ; 1026 ; 1028 ]

20. If the mappings $f : \Re \rightarrow \Re$, $g : \Re \rightarrow \Re$ are defined by $f(x) = x^2$, $g(x) = x + 3$, then $(f \circ g)(-2) =$ .............
[ 1; 2; 3; 4 ]

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

APR 11 2017

Level : B. E./B. Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MCSC 201
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. State the "Division Algorithm Theorem" for integers. If GCD (a, c) = 1 and c | a b, then prove that c | b. Also, find the GCD (d) of the integers a = 1986 and b = 1768 and then express d as s a + t b, for some integers s and t. [1 + 2 + 4]

2. Define an equivalence relation on a set ? Let R be an equivalent relation on a set A then for each a, b in A, prove that a R b if and only if R(a) = R(b).Also, let A = ***Z***, the set of integers and let n = 3, then verify that the relation
R = {(a, b) ∈ A × A : a ≡ b(mod 5)} is an equivalence relation on A and then compute its quotient set. [1 + 2 + 3 +1]

**OR**

Define a graph with example. If a graph has more than two vertices of odd degree, then prove that there can be no Euler path in G [2 + 2 + 3]

3. Define a group and its subgroup. If f is a homomorphism from group (G, ∗) onto group (G', ∗') with respective identities e and e', then show that (i) f(e) = e', and (ii) for any subgroup H of G, f(H) = {f(h) : h ∈H} is a subgroup of G'. [2 + 2 + 3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Let A = {1, 2, 3, 4, 5, 6} and $p = (1 \quad 2 \quad 4 \quad 3)$ be a permutation of A. Then (i) compute $p^2$ and $p^{-1}$, and (ii) find the period of $p$.

5. Find an explicit formula for the sequence defined by $d_n = 4d_{n-1} + 5d_{n-2}$ with initial conditions $d_1 = 2$ and $d_2 = 6$. Also, verify the answer.

6. Let A = {1, 2, 3} and consider two relations $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 3)\}$ and $S = \{(1, 1), (1, 2), (2, 2), (3, 2), (3, 3)\}$, then verify that
$$(SoR)^{-1} = R^{-1} o S^{-1}.$$

7. If $(L_1, \le)$ and $(L_2, \le)$ are lattices, then prove that $(L, \le)$ is a lattice where $L = L_1 \times L_2$, and the partial order $\le$ of $L$ is the product partial order.

**OR**

Show that the power set $P(A)$ defined on A = {1, 2, 3} with set inclusion ( $\subseteq$ ) is a Lattice. Also, draw its Hasse diagram.

8. Use the Principle of Mathematical Induction to prove that $1+2^n < 3^n$ for $n \geq 2$.

9. Find the transitive closure of the relation S = {(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3)} defined on the set A = {1, 2, 3, 4}.

## SECTION "E"

[5 Q. × 2 = 10 marks]

10. If $A \subseteq C$ and $B \subseteq D$, then prove that $A \times B \subseteq C \times D$.

11. Use the method of contradiction to prove that for an integer n if $n^2$ is odd then n is odd.

12. Find the number of ways of arranging 4 white, 3 green and 5 red balls in a row.

13. Construct the tree of the algebraic expression:

$$((3 \times x) + (3 - (6 \times x))) + (x - (3 \times 13)).$$

14. Construct the multiplication table for $(\boldsymbol{Z}_6, \oplus)$, the symbols have their usual meanings.