KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.E./B.Sc.
Year : II

Course : MCSC 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 09 JUL 2023

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. The complete graph K_7 is _____ regular.

2. The cardinality of the range of characteristic function is _____.

3. A path in a connected graph is _____, if all the edges appear exactly once.

4. The truth value of the statement: $x^2 = y^2 \Leftrightarrow x = y, \forall x, y \in \mathbb{R}$ is_____.

5. The product of odd and even permutations is _____.

6. The compound propositions p and p are logically equivalent if _____ is a tautology.

7. If x and y are the positive numbers less than one, then the maximum value of $\lfloor x + y \rfloor$ is_____.

8. The order of an element i in a group $(\{\mathbf{1}, \boldsymbol{i}, -\mathbf{1}, -\boldsymbol{i}\}, \times)$ is_____.

9. If $(M_R)^2_{\odot} = M_R$ for a relation R on a set A, then R is _____.

10. If $(A, \leq)$ is a poset, then elements $a, b \in A$ are _____ if $a \leq b$ or $b \leq a$.

SECTION "B"
[10Q. × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. If $A = \{p, q, r, s\}$, then $|P(A)|$ is _____, where $P(A)$ is the power set of A.
[4; 8; 16; 32]

12. The in-degree of 3 in the digraph of the relation $R = \{(1,2),(1,3),(2,1),(2,4),(3,4),(4,3),(4,1)\}$ on $A = \{1,2,3,4\}$ is _____.
[0; 1; 2; 3]

13. The number of edges in a Hasse diagram of D_{20} is _____.
[6; 7; 8; 9]

14. A monoid (M,$*$) is called group if _____, where e is the identity element in M under the operation $*$.
[$a * b = b * a = e$; $a * (b * c) = (a * b) * c$;
$a + b = b + a$; $a + (b + c) = (a + b) + c$]

15. If a set A has 8 elements and a set B has 10 elements, how many relations are there from A to B?_____.
[2^{72}; 2^{80}; 80; 2^{99}]

16. If L is a lattice such that $a \vee b = b$ for each $a, b \in L$, then we have_____.
[$a < b$; $b < a$; $a \leq b$; b<= a]

17. _____ cannot be the degree sequence of a graph.

[1, 2, 3, 4; 1, 2, 3, 5; 1, 2, 4, 5; 2, 3, 4, 5]

18. If f is mod-11 function, then $f(334) =$_____.
[1; 4; 7; 10]

19. The negation of $p \vee q$ is _____.
[$\sim p \wedge \sim q$; $\sim q \vee \sim p$; $p \Longrightarrow q$; $q \Longrightarrow p$]

20. Which of the following can only be used in disproving the statements?

_____

[Direct proof; Contrapositive proof;
Counter-example; Mathematical induction]

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Time : 2 hrs. 30 mins.

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Semester : I
F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Define equivalence relation. Prove that the relation $R = \{(x, y): x \equiv y(mod\ n)\}$ is an equivalence relation. Also, find an equivalence relation of a set $A = \{1,2,3,4\}$ determined by the partition $P = \{\{1,2\},\{3,4\}\}$. [1+4+2]

2. Define group and subgroup with examples. Prove that a set $S = \left\{1, \frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}i}{2}\right\}$ with $i = \sqrt{-1}$ forms an abelian group under usual multiplication operation. [2+5]

OR

Define general linear group and special linear group. Show that the set of all 2×2 matrix of the form $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}, ad - bc \neq 0\right\}$ is a group under usual matrix multiplication. [2+5]

3. Define cyclic permutation and transposition. Prove that every cyclic permutation can be express as a product of transpositions. Express the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$ as a product of transpositions [1+4+2]

SECTION "D"
[6Q. × 4=24 marks]

4. Define a Boolean matrix. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$, compute $A \vee B, A \wedge B$ and $A \odot B$ where symbols have their usual meaning. [1+3]

5. Prove the following using mathematical induction on $n \geq 0$.
For any real number $x > -1$, prove that $(1 + x)^n \geq 1 + nx$.

6. Construct two random graph with six and ten vertices and verify that the number of odd vertices in the graph is even.

7. Draw a digraph of a relation R from the following matrix representation. [2]

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Also, find R^{∞} from the digraph. [2]

8. Define partial order relation on a non-empty set. Prove that the digraph of partial order relation has no cycle of length greater than one. [1+3]

OR

Show that the relation $S = \{(m, n) \in \mathbb{Z}^+ \times \mathbb{Z}^+ : m \text{ divides } n\}$ is a partial order relation.

9. What do you understand by valid argument? Determine whether the following arguments is valid or not.

If taxes are lowered, then income rises.
Income rises.

∴Taxes are lowered.

SECTION "E"
[5Q. × 2=10 marks]

10. Find the GCD of 540 and 504 using Euclidean algorithm.

11. Find the period of the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ defined on $A = \{1, 2, 3, 4, 5, 6\}$.

12. Is the relation $' \leq '$ on $\mathbb{Z}$ anti-symmetric? Justify your answer.

13. Prove that the inverse of an element in a group $(G, *)$ is unique.

14. Define Boolean algebra. Is D_{20} a Boolean algebra? Justify your answer.