KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

Level : B.E./ B.Sc.
Year : II
Exam Roll No. :

Time: 30 mins.

Course : MCSC 201
Semester : I
F. M. : 20

Registration No.:

Date 05 MAR 2019

SECTION "A"
[10 Q. × 1=10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. If a_n denote the nth term of a sequence $1, 3, 5, 7, \cdots$. Then the explicit formula for this sequence is $a_n =$ ______________________.

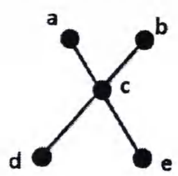
2. Suppose p and q are two statements. Then the statement $p \Rightarrow q$ is logically equivalent to the statement ______________________.

3. Let $A = B = \{1, 2, 4, 8\}$. Then the R-relative set of the element 4, that is, $R(4) =$ ______________.

4. Let $f : A \to B$ be a function, and 1_B be an identity function on B. Then $1_B \circ f =$ ______________.

5. Suppose $\Delta = \{ (a, a) | a \in A \}$ be an equality relation on A, and R be an arbitrary relation on A such that ____________________, then R is a reflexive relation on A.

6. The degree of the vertex c in the following graph



is ________________.

7. Let G and G_1 be two groups with their respective identity elements as e and e_1, and consider a homomorphism $f : G \to G_1$. Then $f(e) =$ ______________________.

8. Consider a set of natural numbers $\mathbb{N}$, and define a relation R on $\mathbb{N}$ be $a\, R\, b \Rightarrow a \leq b$ for all $a, b \in \mathbb{N}$. Then the least element of $\mathbb{N}$ is ______________________ under the relation R.

9. If e is an identity element of a monoid $(M,*)$. Then e is ______________________.

10. If $p_0 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ and $p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ be two permutations on a set $A = \{1,2,3\}$. Then $p_0 \circ p_1 =$ ______________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. The number ____________________ is identity in the semigroup $(\mathbb{Z}, +)$, where $\mathbb{Z}$ is the set of integers.
[-1; 0; 1; ∞]

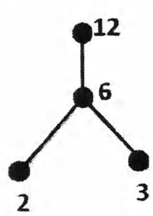
12. Let D_n denotes the collection of all positive integer divisors of n. Then ______________ is the Boolean algebra.
[D_3; D_{20} ; D_{40} ; D_{210}]

13. Suppose $A \subseteq U$, and $f_A: U \to \{0, 1\}$ is a characteristic function of A. Then $f_A(u) =$ __________, if $u \notin A$.
[-1; 0; 1; ∞]

14. The base 4 representation of the number 115 is ____________________.
[$(302)_4$; $(202)_4$; $(303)_4$; $(103)_4$]

15. If each vertex of a graph has the same degree as every other vertex, then the graph is said to be a ____________________ graph.
[connected; complete; discrete; regular]

16. Consider the following Hasse diagram of a Poset



The LUB({2,3}) = ____________________.
[6; 12; 3; does not exist]

17. Suppose M_R be the matrix relation of a transitive relation R, then $M_R \odot M_R =$ ______________.
[$M_{R^{-1}}$; M_R; M_{R^2} ; M_{R^∞}]

18. If 6 colors are used to paint 49 cars, then at least ____________________ cars will be of the same color.
[6; 7; 8; 9]

19. A semigroup that has an identity element is called a ____________________.
[groupoid; sub-semigroup; monoid; group]

20. Let a be an integer and let p be a positive integer. If $p|a$, then $p =$ ____________________.
[LCM(a, p); GCD(a, p) ; LUB(a, p) ; GLB(a, p)]

KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

05 MAR 2019

Level : B.E./B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MCSC 201
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define a partially ordered set. Let (A, R_A) and (B, R_B) be two partially ordered sets. Then, prove that $(A \times B, R)$is a partially ordered set with product partial order R defined by

$$(a,b)R\,(a',b') \Leftrightarrow a\; R_A a' \text{ and } b\; R_B b'$$

where $a, a' \in A$ and $b, b' \in B$. Also, let $A = \mathbb{R}$, be the set of real numbers, and define $a\; R\; a' \;\Leftrightarrow a \leq a'$, where $\leq$ is the usual less than equal to in $\mathbb{R}$. Show that (A, R) is a partially ordered set. [1 + 4 + 2]

OR

Define a Lattice. If (L_1, R_1) and (L_2, R_2) are lattices, then prove that $(L, R)$is a lattice, where $L = L_1 \times L_2$, and the partial order of L is the product partial order. [1 + 6]

2. Let $a, b \in \mathbb{Z}^+$, the set of positive integers, then prove that GCD $(a, b) \times$ LCM $(a, b) = ab$. Verify this result for $a = 540$ and $b = 504$. [5 + 2]

3. Define converse, inverse and contrapositive of an implication $p \Rightarrow q$. When an argument is said to be valid? Show the validity of the following arguments

If you invest in the stock market, then you will get rich.
If you get rich, then you will be happy.

∴ If you invest in the stock market, then you will be happy. [2 + 5]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Let $\mathbb{Z}^+$ be a set of positive integers, and define an operation on $\mathbb{Z}^+$ by $a * b = a + b + ab$ for all $a, b \in \mathbb{Z}^+$. Show that $(\mathbb{Z}^+, *)$ is a semigroup.

OR

Let G be a group, and $a, b, c \in G$. Prove that $(i)\; ab = ac \;\Rightarrow bc,\; (ii)(ab)^{-1} = b^{-1}a^{-1}$, where the symbols have their usual meanings.

5. Draw a picture of the graph $G = (V, E, \gamma)$, where $V = \{a, b, c, d, e\},\; E = \{\, e_1, e_2, e_3, e_4, e_5, e_6 \,\}$, and $\gamma(e_1) = \gamma(e_5) = \{a, c\}, \gamma(e_2) = \{a, d\}, \gamma(e_3) = \{e, c\}, \gamma(e_4) = \{b, c\}$ and $\gamma(e_6) = \{e, d\}$.

6. Let $A = \{1, 2, 3, 4, 5, 6\}$, and $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation on A.

(a) Write p as a product of disjoint cycles.

(b) Compute p^{-1}.

(c) Compute p^2.

(d) Find k such that $p^k = 1_A$, the identity element on A.

7. Define Least upper bound and Greatest lower bound. Let $B = \{c, d, e\}$, then find, if they exists

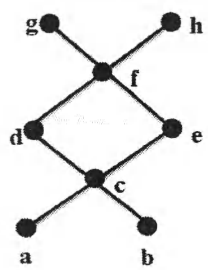
(a) All upper bounds of B.

(b) All lower bounds of B.

(c) The least upper bound of B.

(d) The greatest lower bound of B

from the Poset as shown in Figure below:



8. Let R be a relation on $A = \{1, 2, 3, 4\}$ given by $R = \{(a, b),\ (a, d),\ (b, a),\ (b, c),\ (b, d),\ (c, b),\ (d, a),\ (d, b)\}$.

(a) Determine M_R, the matrix relation of R.

(b) Determine whether R is reflexive, irreflexive and symmetric with reason.

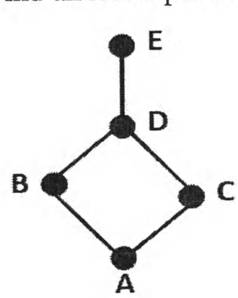
9. Consider $A = \{1, 2, 3, 4, 8\}$ and $R \subseteq A \times A$ be a relation on A defined by $a\ R\ b \Leftrightarrow a + b \leq 9$ for $a, b \in A$. Find the relation R, and its digraph.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Let $\mathbb{R}$ be the set of real numbers, and a binary operation $*$ on $\mathbb{R}$ be defined by $a * b = \frac{ab}{2}$. Find the identity element of $\mathbb{R}$ under the binary operation $*$, if it exists.

11. Find an Euler path in the graph given below:



12. Consider the Boolean matrices $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Find the Boolean product $A \odot B$.

13. If p and q are two statements, show that $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$.

14. Let $a, b, c \in \mathbb{Z}^+$. If $a|b$ and $a|c$, then prove that $a|(b + c)$.