KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

Marks Scored:

Level : B.E./B.Sc.
Year : II

Course : MCSC 201
Semester : II

Exam Roll No.:

Time: 30 mins.

F. M. : 10

Registration No.:

Date 07 DEC 2023

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. Let $A = \{a, b, c\}$ be a set with three elements. Then the number of even permutations is _____.
2. The product of two odd permutations is _____.
3. let $|A| = n$ and $|B| = m$. Then number of relations from A to B is _____.
4. If A and B are two non-empty sets satisfying $A \cup B = A \cap B$, then A and B are _____.
5. The fifth term of the sequence given by recursive relation is $a_1 = -3, a_n = -2a_{n-1} + 1$ is _____.
6. If h is mod-6 function, then $h(-19) =$_____.
7. The degree of a vertex in a complete graph K_n is _____.
8. A path in a connected graph is _____, if all the nodes appear exactly once.
9. If the truth value of a proposition is always true, then the proposition is _____.
10. The order of an element $-i$ in a group $(\{1, i, -1, -i\}, \times)$ is_____.

SECTION "B"

[10Q. × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The loop in every vertex of a digraph indicates that the relation is _____
[symmetric; reflexive; transitive; equivalent]

12. If $a^7 = e, a \neq e$ for some element a in a group, where e is the identity element, then 7 is the _____ of a.
[period; degree; order; size]

13. The contrapositive of $p \Longrightarrow q$ is _____.
[$q \Longrightarrow p$; $\sim p \Longrightarrow \sim q$; $\sim q \Longrightarrow \sim p$; $\sim q \Longrightarrow p$]

14. Suppose a linear Hasse diagram of a poset has three nodes. Then actual number of edges in a digraph representation of that partial order relation is _____.
[6; 7; 8; 9]

15. The out-degree of 2 in the digraph of the relation
$R = \{(1,2),(1,3),(2,1),(2,4),(3,4),(4,3),(4,1)\}$ on $A = \{1,2,3,4\}$ is _____.
[0; 1; 2; 3]

16. A monoid is called group if _____
[$a * b = b * a = e$; $a * (b * c) = (a * b) * c$;
$a + b = b + a$; $a + (b + c) = (a + b) + c$]

17. The number of edges in a complete graph K_8 is _____.
[64; 56; 28; 15]

18. The period of the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 2 & 6 & 1 \end{pmatrix}$ is _____.
[2; 4; 6; 8]

19. Let $R = \{(a,b): a = b - 2, b > 6\}$ be a relation on $\mathbb{N}$. Which of the following belongs to R?
[$(2,4)$; $(3,5)$; $(6,8)$; $(8,7)$]

20. The proposition $(p \Longrightarrow q) \wedge (q \Longrightarrow p)$ is a _____.
[tautology; contradiction; contingency; absurdity]

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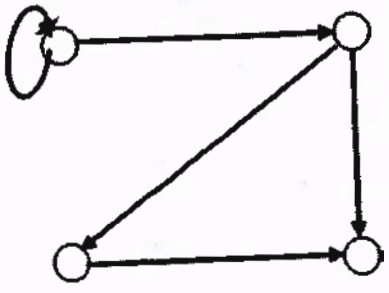
07 DEC 2023

Level : B.E./B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MCSC 201
Semester : I
F. M. : 55

SECTION "C"
[3Q × 7 = 21 marks]

1. Define path in a connected graph. Give name to the nodes of the graph alongside and write down the matrix representation of the digraph below. Also, find the number of paths of length two from a vertex to every other vertex. [1+2+4]



2. Prove that a set $S = \{1, -1, i, -i\}$ form an abelian group under usual multiplication operation. Also, find the order of each element. [4+3]

3. Define poset. Let $P(A)$ be the power set of a set $A = \{a, b, c\}$. Prove that $(P(A), \subseteq)$ is a poset. Also, draw the Hasse diagram of $P(A)$ with partial order relation $' \subseteq '$. [1+3+3]

OR

Let $R = \{(1,2), (1,3), (2,1), (2,2), (2,3)\}$ and $S = \{(1,3), (2,3), (3,2), (3,3)\}$ be two relations on a set $A = \{1, 2, 3\}$ Then verify the following conditions: [3+4]

a. $M_{R \cup S} = M_R \vee M_S$
b. $M_{R^2} = M_R \odot M_S$

SECTION "D"
[6Q × 4 = 24 marks]

4. Find the GCD of $a = 273$ and $b = 93$ using Euclidean algorithm. Also, verify that $GCD(a, b) = GCD(a \pm b, a)$. [2+2]

5. Write the implication in symbolic form and state its inverse, converse and contrapositive of "If I have enough money, then I will buy a car and I will buy a house."

6. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p_1 = (4, 1, 3, 5)$, $p_2 = (5, 6, 3)$ be two cyclic permutations on A. Find $p_1 \circ p_2$ and $p_2 \circ p_1$. In which case, the product of permutations is commutative? [3+1]

7. What do you mean by a Boolean algebra? Show that D_{30} is a Boolean algebra. [1+3]

8. Prove by mathematical induction that if $A_1, A_2, \ldots, A_n$ are n sets, then

$$\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A}_i, n \geq 2$$

9. Let $A = \{1,2,3,4\}$ and $f = \{(1,1),(2,3),(3,4),(4,2)\}$ be a relation on A. Determine whether f is a function or not. If it is a function, is it one-to-one or onto or both or neither? [2+2]

OR

Draw a digraph of a relation R from the following matrix representation: [2]

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Also, find R^{∞} from the digraph. [2]

SECTION "E"

[5Q × 2=10 marks]

10. Prove that if $*$ is an associative operations and x has $*$ −inverse y, then y is unique.

11. Show that if 7 colors are used to print 85 T-shirts for promotional event, at least 13 T-shirts will be of same color.

12. Show that $A \oplus B = (A - B) \cup (B - A)$.

13. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation defined on $A = \{1,2,3,4,5,6\}$. Determine whether or not p is even.

14. Find the relation R on $\{2,3,4,5,10,20\}$ such that $R = \{(x,y): x \text{ divides } y\}$.