KATHMANDU UNIVERSITY
End Semester Examination [C]
April/May, 2023

Marks Scored:

Level : B.E./B.Sc.
Year : II
Course : MCSC 201
Semester : I

Exam Roll No.:
Time: 30 mins.
F. M. : 20

Registration No.:
Date : 27 APR 2023

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. If f is the mod-11 function, then $f(1232)$ = _____.

2. To prove a theorem means to show that the implication $p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q$ is a(n) _____.

3. Let R be a relation from A to B, and $M_R = [m_{ij}]$ be the matrix representation of R. If R is irreflexive, then m_{ij} = _____ for $i = j$.

4. Let $A = \{1,3,5,7\}$ and $\Omega = \{1,2,\ 3,4,5,6,7,8,9,10\}$ and $f_A: \Omega \to \{0,1\}$ be the characteristic function. Then $f_A(2)$ = _____.

5. $(P(A), \subseteq)$ is a Poset, where $A = \{a,b,c\}$. Then the number of vertices in the Hasse diagram of $(P(A), \subseteq)$ is _____.

6. Let B be a Boolean algebra, and $a, b \in B$. Then $(a \wedge b)'$ = _____.

7. Let $U_n, n \geq 1$ denote a discrete graph. Then the degree of each vertex of U_n is _____.

8. If $(G,\ \Delta)$ be a group, then $(a\ \Delta b)^{-1}$ = _____ for all $a, b \in G$.

9. Two statements p and q are called logically equivalent if $p \Leftrightarrow q$ is a(n) _____.

10. Let $A = \{a_1, a_2, \cdots, a_n\}$ be a finite set with n elements, $n \geq 2$. Then the total number of even permutations = _____.

SECTION "B"
[10 Q. × 1 = 10 Marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. An $m \times n$ matrix whose entries are 0 or 1, is known as a(n) _____ matrix.
[Boolean, adjacent, consistent, permanent]

12. If p is any statement, then $p \wedge \sim p$ is a(n) _____.
[contingency, contradiction, tautology, equivalent]

13. The number of partitions of a set with n elements into k subsets satisfy the recurrence relation $S(n,k) = S(n-1,k-1) + k\,S(n-1,k)$ with initial conditions $S(n,1) = 1$, and $S(n,n) =$ _____.
[0, 1, 2, n]

14. Let $A = \{2,3,4\}$, $B = \{a,b,c,d\}$, and a function f from A to B is defined by $f = \{(2,a),(4,b),(3,c)\}$. Then the function f is _____.
[one-to-one, onto,
one-to-one correspondence, bijection]

15. Let R be a relation on a non-empty set S, and (S,R) is a Poset. Then R is _____.
[irreflexive, symmetric, asymmetric, antisymmetric]

16. Let $A = \{1,2,3,4\}$ and let $R = \{(1,1),(2,2),(3,1),(3,2),(3,3),(4,2),(4,4)\}$ be a relation on A. Then the out degree of 1 is _____.
[1, 2, 3, 4]

17. A complete graph K_n has _____ edges.
[$n(n-1)$, $\frac{n(n-1)}{2}$, $n(n+1)$, $\frac{n(n+1)}{2}$]

18. A group $(G, *)$ is said to be abelian if _____ for all $x, y \in G$.
[$x + y = y + x$, $xy = yx$, $x * y = y * x$, $x * y = x + y$]

19. A graph with only one vertex is called _____.
[empty, trivial, connected, discrete]

20. The explicit formula for the n^{th} term of the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \cdots$ is _____.
[$\frac{1}{2^{n-1}}$, $\frac{1}{2^n}$, $\frac{1}{2^{n+1}}$, $1 - \frac{1}{2^n}$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
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27 APR 2023

Level : B.E./B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MCSC 201
Semester : I
F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. What do you mean by the path of length n in a relation? If R is a relation on $A = \{a_1, a_2, \cdots, a_n\}$, then prove that $M_{R^2} = M_R \odot M_R$. Also, verify it for a relation $R = \{(a,a),(a,b),(a,c),(a,d),(b,c),(c,b),(d,b),(d,c),(d,d)\}$ on set $A = \{a,b,c,d\}$. [1 + 3 + 3]

OR

Let R and S be relations on a set A. Then prove the following
a. If R is reflexive, so is R^{-1}. [2]
b. If R and S are reflexive, then so are $R \cap S$. [2]
c. R is reflexive if and only if $\overline{R}$ is reflexive. [3]

2. Define groupoid, semigroup, monoid, and group. Give an example with the reason that is a monoid but not a group. Let $(S, *)$ and $(T, \triangle)$ be monoids, then show that $S \times T$ is also a monoid. [2 + 2 + 3]

3. What are the conjunction, disjunction, and implication of any two statements? Show that the compound statement $\big((p \Rightarrow q) \wedge (q \Rightarrow r)\big) \Rightarrow (p \Rightarrow r)$ is a tautology. [3 + 4]

SECTION "D"
[6 Q. × 4 = 24 Marks]

4. Find the greatest common divisor d of $a = 34$ and $b = 58$, and write d as $s\,a + t\,b$, where s and t are integers.

OR

Let a be an integer and let p be a positive integer. Prove that if $p \mid a$, then $p = \text{GCD}(a,p)$.

5. If $f: A \to B$ and $g: B \to C$ are one-to-one functions, then show that $g \circ f$ is one-to-one.

6. Determine the Hasse diagram of the relation on $A = \{1,2,3,4,5\}$ whose matrix is shown

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

7. Define the complete graph. Also, draw the complete graph on seven vertices.

8. What do you mean by linear order in a Poset? Let R be a relation on $A = \mathbb{R}$, the set of real numbers defined by $a\,R\,b$ if and only if $a \leq b$. Then show that R is a linear order.

9. Use mathematical induction to prove that $1 + 2^n < 3^n$ for $n \geq 2$.

SECTION "E"

[5 Q. × 2 = 10 Marks]

10. Let $A = \{1, 2, 3, 4\}$ and let $R = \{ (1, 2), (1, 3), (4, 2)\}$ be a relation on A. Find M_R.

11. Determine whether the binary operation $*$ defined on $\mathbb{R}$, the set of real numbers, by $a * b = \frac{ab}{3}$ is associative.

12. Draw a picture of the graph $G = (V, E, \gamma)$, where $V = \{1, 2, 3, 4\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$ and γ be defined by $\gamma(e_1) = \gamma(e_5) = \{1, 2\}$, $\gamma(e_2) = \{4, 3\}$, $\gamma(e_3) = \{1, 3\}$, and $\gamma(e_4) = \{2, 4\}$.

13. Write a recursive formula for the sequence 2, 5, 7, 12, 19, 31, …

14. Draw a truth table of the compound statement $\sim p \wedge \sim q$.