KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc.
Year : III

Course : MATH 327
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 10

Registration No.:

Date :

SECTION "A"

[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. If $gcd(a,b) = d$, then $gcd(a/d, b/d) =$ ______________________.

2. Let $d = \gcd(a,b)$ and $n \in \mathbb{N}$. If $d|c$ and (x_0, y_0) is a solution of Diophantine equation $ax + by = c$, then all the integers solution is given by ______________________.

3. If a and b are relatively prime positive integers, then the arithmetic progression $a, a+b, a+2b, a+3b \ldots contains$ ______________ primes.

4. A number-theoretic function f is said to be multiplicative if ______________, whenever $\gcd(m,n) = 1$.

5. Let p be a prime and m a positive integer, then $\varphi(p^m) =$ ______________.

6. If f(n) is an arithmetic function, then $F(18) \sum_{d|18} f(d) =$ ______________.

7. 561 is a base ______________ pseudo-prime.

8. $\left[\frac{53}{59}\right] =$ ______________, the sign [ ] has usual meaning.

9. The set of reduced residue system modulo 8 is ______________.

10. Let p be a prime and suppose that p does not divide a. Then ______________.

SECTION "B"

[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. **(DO NOT TICK THE ANSWER)**.

11. If $a, b \in \mathbb{Z}$ and $b \neq 0$, then $\exists$ a unique pair of integer q and r, such that $a = bq + r$ where ______________.
    a. $0 \leq r \leq |b|$ b. $0 \leq r < |b|$ c. $0 > r > |b|$ d. $0 \leq r \leq b$

12. Let $d = (a, m)$ then the congruence $ax \equiv b(modm)$ has a solution iff ______________.
    a. $b|d$ b. $m|d$ c. $d|b$ d. $a|d$

13. Which theorem state that If p is a prime, then $(p-1)! \equiv -1(modp)$? ____________.

a. Euler's Theorem
b. Wilson's Theorem
c. Fermat's Little Theorem
d. Dirichlet's Theorem

14. Which name matches statement if $a|bc$ and $(a,b) = 1$ then $a|c$?________________.

a. Euclid's Lemma
b. Fermat's Theorem
c. Division Algorithm
d. Euclidean Algorithm

15. An integer n is called a pseudo-prime if ________________________________.

a. $n|2^n - 2$
b. n is composite and $n|2^n$
c. n is prime and $n|2^n - 2$
d. n is composite and $n|2^n - 1$

16. If $p, q_1, q_2, \ldots\ldots, q_n$ are all primes and $p|q_1q_2q_n$, then ________________________.

a. $p = q_k$ for some k
b. $p = 2$
c. $q_k = 2$ for some k
d. $p|q_k$ for some k

17. $\sigma(n) = n + 1$ if and only if ________________________________.

a. n is an odd number
b. n is an even number
c. n is a prime number
d. n is a neither odd nor even

18. The number of primitive roots for 19 is _______.

a. 3
b. 4
c. 6
d. 8

19. The inverse of 37 modulo 49 is ________.

a. 23
b. 4
c. 12
d. 6

20. The order of 2 modulo 7 is _______.

a. 1
b. 3
c. 2
d. 4

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Level : B.Sc.
Year : III
Time : 2 hrs. 30 mins.

Course : MATH 327
Semester : II
F. M. : 50

SECTION "C"
[6Q. × 7 = 42 marks]

1. Prove that if d is a common divisor of a and b then $d = \gcd(a, b)$ if and only if $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$. We have an unknown number of coins. If you make 77 strings of them, you are 50 coins short; but if you make 78 strings, it is exact. How many coins are there? [3+4]

2. Define a linear congruence $modulo\, n$. Prove that if $ca \equiv cb(modn)$, then $a \equiv b(mod \frac{n}{d})$, where $d = \gcd(c, n)$. Find the solution of the given system of congruence. [1+2+4]

$$6x + 8y \equiv 10(mod13)$$
$$6x + 15y \equiv 21(mod13)$$

OR

Define congruence modulo. State Chinese Remainder theorem. A certain integer between 1 and 1200 leaves the remainders 1, 2, 6 when divided by 9, 11, 13 respectively. What is the integer?

3. State and prove Wilson's Theorem. Make a concrete example that should help to clarify the proof of Wilson's theorem. [1+3+3]

4. Define the order of integer modulo n. Find the order of integers 2 Modulo 17. Make the table of the indices for a primitive root of 11 and then solve the given congruence $7x^3 \equiv 3mod(11)$. [1+2+4]

5. Define Number-theoretic function. Prove that σ, τ, μ are Number-theoretic functions, these symbols having their usual meaning. [1+6]

6. Define RSA Cryptosystem. Write RSA Algorithm. Write an example performing encryption and decryption a message using RSA algorithm. [1+3+3]

SECTION "D"
[4Q. × 2 = 8 marks]

7. Verify that the integers 1949 and 1951 are twin prime.

8. Use Fermat's theorem to verify that 17 divides $11^{104}+1$.

9. Verify that the equality $\phi(n) = \phi(n + 1) = \phi(n + 2)$ holds when $n = 5186$.

10. Show that $85 = 5.17$ is a base 4 pseudo-prime but not a base 2 pseudo-prime.

Table — EPEG-318

25 JUN 2023

Single Phase Half Wave Diode Rectifier:

With R Load:

- **RMS value of output voltage**

  $V_{0\,rms} = \frac{V_m}{2}$

  V_m= Maximum value of source voltage (V_s)

- **Average value of output voltage,**

  $V_{0\,DC} = \frac{V_m}{\pi}$

- Average value of output voltage,
- Power delivered $P = I^2_{0\,rms} \cdot R$

  I_0 rms = rms value of load current = $\frac{\text{Power delivered to load}}{\text{Input } V_A}$

- Input power Factor = $\frac{V_{0\,rms} \cdot I_{0\,rms}}{V_s - I_{0\,rms}} = 0.707$

With L load:

- **Output current** $I_0 = \frac{V_m}{\omega L}(1 - \cos \omega t)$
- Maximum value of current $I_0 = \frac{2X_m}{\omega L}$
- Average value of current $I_0 = \frac{I_{max}}{2}$
- RMS value of fundamental current

  $I_{1\,rms} = \frac{I_0}{\sqrt{2}}$

- Output voltage $V_0 = V_m \sin \omega t = V_s$
- Average value of current voltage $V_0 = 0$

With C Load:

- Output voltage $V_0 = V_m \sin \omega t = V_s = V_c$
- Diode voltage $V_D = V_m (\sin \omega t - 1)$
- Output current $I_0 = \omega c\, V_m \cos \omega t$
- Average value of diode voltage $V_D = V_m$
- RMS value of diode voltage

  $V_{rms\,D} = 1.225\, V_m$

Single-Phase Full wave Mid-point Diode Rectifier:

Average output voltage,

$$V_0 = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t)$$

$$\boxed{V_0 = \frac{2V_m}{\pi}}$$

Average output current,

$$i_0 = \frac{V_0}{\pi}$$

Rms value of load voltage,

$$V_{0(rms)} = \sqrt{\left[\frac{1}{2}\int_0^{\pi} V_m^2 \sin \omega t \, d(\omega t)\right]}$$

$$\boxed{V_{0(rms)} = \frac{V_m}{\sqrt{2}} = V_s}$$

Rms value of load current,

$$I_{0(rms)} = \frac{V_s}{R}$$

Power delivered to load = $V_{0(rms)}\, I_{0(rms)}$

$\therefore$ Input power factor = $\frac{V_{0(rms)} I_{0(rms)}}{V_s I_{0(rms)}}$

Single-Phase Full wave Diode Bridge Rectifier:

Average value of diode current,

$$I_D = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t . d(\omega t) = \frac{I_m}{\pi}$$

Rms value of diode current,

$$I_{D(rms)} = \sqrt{\left[\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

$$\boxed{I_{D(rms)} = \frac{I_m}{2}}$$

Single phase half wave-controlled rectifier with R Load

Average output voltage,

$$V_0 = \frac{V_m}{2\pi}(1 + \cos \alpha)$$

Average output current,

$$I_0 = \frac{V_m}{2\pi R}(1 + \cos \alpha)$$

Rms value of output voltage,

$$V_{or} = \frac{V_m}{2\sqrt{\pi}}\left[(\pi - \alpha) + \frac{1}{2}\sin 2\alpha\right]^{1/2}$$

Commutation time or turn off time of the thyristor,

$$t_c = \frac{\pi}{\omega} \text{ sec}$$

Input power factor of the converter,

$$\text{p.f.} = \frac{V_{or} . I_{or}}{V_s . I_{or}} = \frac{1}{\sqrt{2\pi}}\left[\pi - \alpha + \frac{1}{2}\sin 2\alpha\right]^{1/2}$$

Single phase half wave-controlled rectifier with RL load:

Average output voltage,

$$V_0 = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

Average output current,

$$I_0 = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta)$$

Rms value of output voltage.

$$V_{or} = \frac{V_m}{2\sqrt{\pi}}\left[(\beta-\alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{\frac{1}{2}}$$

Commutation time or turn off time for the thyristor,

$$t_c = \frac{2\pi - \beta}{\omega}\text{sec}$$

Single phase half wave-controlled rectifier with RLE load:

The minimum value of firing angle at which thyristor can be triggered is

$$\theta_1 = \sin^{-1}\left(\frac{E}{V_m}\right)$$

Average output current,

$$I_0 = \frac{1}{2\pi R}\left[V_m(\cos\alpha - \cos(\gamma + a)) - E\gamma\right]$$

Average output voltage,

$$V_0 = \frac{1}{2\pi}\left[V_m(\cos\alpha - \cos\beta) + E(2\pi + \alpha - \beta)\right]$$

Input power factor,

$$p.f. = \frac{(I_{or}^2 R + EI_0)}{V_s I_{or}}$$

Commutation time or turn off time of thyristor,

$$t_c = \frac{2\pi - \beta}{\omega}\text{sec}$$

Bridge Inverter: Bridge circuits are commonly used in DC-AC conversion. Moreover, an output transformer is not essential in a bridge circuit.

1φ Half Bridge Inverter - The output voltage volt $V_0 = \sum_{n=1,3,5...}^{\infty} \frac{2V_s}{n\pi}\sin n\omega t$

1φ Full Bridge Inverter- The output voltage

$$V_0 = \sum_{n=1,3,5...}^{\infty} \frac{4V_s}{n\pi}\sin n\omega t$$

Where, n = order of harmonic

ω=2πf, is frequency of the output voltage in red/sec

Key points:

- The load impedance (Z_n) at frequency

$$n_f Z_n = \left[R^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2\right]^{1/2}$$

- Phase angle, (ϕ_n), $\phi_n = \tan^{-1}\frac{\left[n\omega L - \frac{1}{n\omega C}\right]}{R}$ red

- Output current or load current at the instant of commutation $I_0 = I_s = \frac{V_0}{Z_n}$ or $\omega t = \pi$ rad

- Fundamental load power

$(P_{01})\ P_{01} = I_{01}^2 R = V_{01} \cdot I_{01}\cos\phi_1$

Three Phase Half Wave Diode Rectifier:

The peak inverse voltage (PIV) = $\sqrt{3}V_{mp}$ for each of the three diode D_1, D_2 and D_3. The average output voltage.

$$V_0 = \frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_{mp}\sin\omega t\, d(\omega t)$$

$$= \frac{3\sqrt{3}}{2\pi}V_{mp}$$

$$= \frac{3\sqrt{6}}{2\pi}V_{ph} \qquad \left(\because V_{mp} = \sqrt{2}V_{ph}\right)$$

$$= \frac{3}{2\pi}V_{ml} \qquad \left(\because V_{ml} = \sqrt{3}V_{mp} = \sqrt{6}V_{ph}\right)$$

Rms value of output voltage,

$$V_{0(rms)} = \sqrt{\left[\frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} V_{mp}^2 \sin^2\omega t\, d(\omega t)\right]}$$

$$\boxed{V_{0(rms)} = 0.84068\ V_{mp}}$$

Three-Phase Mid-point 6-Pulse Diode Rectifier:

Average output voltage,

$$V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{mp}\sin\omega t\, d(\omega t)$$

$$\boxed{V_0 = \frac{3V_{mp}}{\pi}}$$

Rms value of output voltage,

$$V_{0(rms)} = \sqrt{\left[\frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3}(V_{mp}\sin\omega t)^2 d(\omega t)\right]}$$

$$\boxed{V_{0(rms)} = 0.9558\ V_{mp}}$$

THREE PHASE DIODE BRIDGE RECTIFIER

Average Value of load voltage,

$$V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{ml}\sin\omega t\, (d\omega t)$$

$$\boxed{V_0 = \frac{3V_{ml}}{\pi}}$$

Rms value of output voltage,

$$V_{0(rms)} = \sqrt{\left[\frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{ml}^2\sin^2\omega t\, d(\omega t)\right]}$$

$$\boxed{V_{0(rms)} = 0.9558\ V_{ml}}$$

THREE PHASE HALF-WAVE-CONTROLLED RECTIFIER:

If V_{mp} is the peak value of phase voltage, the average value of the output voltage,

$$V_0 = \begin{cases} \frac{3\sqrt{3}}{2\pi} V_{mp} \cos\alpha & ;\text{for } 0 < \alpha < 30^\circ \\ \frac{3}{2\pi} V_{mp} \left(1 + \cos(\alpha + 30^\circ)\right) & ;\text{for } 30^\circ < \alpha < 150^\circ \end{cases}$$

If V_{ml} is the peak value of line voltage, average output voltage,

$$V_0 = \frac{3V_{ml}}{\pi} \cos\alpha$$

If I_0 is the load current, average value of source current,

$$I_s = I_0\sqrt{\frac{2}{3}}$$

Average value of thyristor current,

$$I_s = I_0\sqrt{\frac{1}{3}}$$

Three phase half-wave converter drive

1. $V_m = \sqrt{2}V_s$
2. Average value of output voltage $V_o = \frac{3V_m}{2\pi}\cos\alpha$ [For $0 \le \alpha \le \pi$]
3. RMS value of phase or line current $I_{sr} = I_a\sqrt{\frac{1}{3}}$
4. Average thyristor current $I_{TA} = I_a\frac{1}{3}$
5. RMS thyristor current $I_{TR} = I_a\sqrt{\frac{1}{3}}$
6. Input PF $= \frac{V_o * I_a}{\sqrt{3} * V_s * I_{sr}} = \frac{3}{\pi\sqrt{2}}\cos\alpha$

Three phase Full-wave converter drive

1. Average value of armature voltage $V_o = \frac{3V_m}{\pi}\cos\alpha$ [For $0 \le \alpha \le \pi$]
2. Average value of field voltage $V_f = \frac{3V_m}{\pi}\cos\alpha_f$ [For $0 \le \alpha_f \le \pi$]
3. RMS value of line current $I_{sr} = I_a\sqrt{\frac{2}{3}}$
4. Average thyristor current $I_{TA} = I_a\frac{1}{3}$
5. RMS thyristor current $I_{TR} = I_a\sqrt{\frac{1}{3}}$
6. Input PF $= \frac{V_O * I_a}{\sqrt{3} * V_s * I_{sr}} = \frac{3}{\pi}\cos\alpha$

Three phase Dual converter drive

1. Average value of output and field circuit voltage $V_f = \frac{3V_m}{\pi}\cos\alpha_f$ for $0 \le \alpha_f \le \pi$

Three phase semi converter drive

1. Average value of armature voltage $V_a = \frac{3V_m}{2\pi}(1 + \cos\alpha_a)$ [For $0 \le \alpha_a \le \pi$]
2. Average value of field voltage $V_f = \frac{3V_m}{2\pi}(1 + \cos\alpha_f)$ [For $0 \le \alpha_f \le \pi$]
3. RMS value of line current $I_{sr} = I_a\sqrt{\frac{2}{3}}$ [For $\alpha < 60^\circ$]
4. RMS thyristor current $I_{TR} = I_a\sqrt{\frac{1}{3}}$ [For $\alpha < 60^\circ$]
5. Average thyristor current $I_{TA} = I_a\frac{1}{3}$ [For $\alpha < 60^\circ$]
6. RMS value of line current $I_{sr} = I_a\sqrt{\frac{180 - \alpha}{180}}$ [For $60^\circ < \alpha < 180^\circ$]
7. RMS thyristor current $I_{TR} = I_a\sqrt{\frac{180 - \alpha}{360}}$ [For $60^\circ < \alpha < 180^\circ$]
8. Average thyristor current $I_{TA} = I_a * (\frac{180 - \alpha}{360})$ [For $60^\circ < \alpha < 180^\circ$]
9. Freewheeling diode RMS current $I_{FR} = I_a * \sqrt{\frac{\alpha - 60}{120}}$ [For $60^\circ < \alpha < 180^\circ$]
10. Freewheeling diode average current $I_{FA} = I_a * \frac{\alpha - 60}{120}$ [For $60^\circ < \alpha < 180^\circ$]
11. Input PF $= \frac{V_a * I_a}{\sqrt{3} * V_s * I_{sr}} = \frac{3}{2\pi}(1 + \cos\alpha)$ [For $\alpha < 60^\circ$]
12. Input PF $= \frac{V_a * I_a}{\sqrt{3} * V_s * I_{sr}} = \frac{\sqrt{6}(1 + \cos\alpha)}{2\pi}\sqrt{\frac{\pi - \alpha}{\pi}}$ [For $60^\circ < \alpha < 180^\circ$]