KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

Marks Scored:

Level : B.E.
Year : III
Course : MATH 326
Semester : I
Exam Roll No. :
Time: 30 mins.
F. M. : 10
Registration No.:
Date : 28 JUL 2024

SECTION "A"
[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. The multiplicative inverse of the complex number $z = 1 - i$ is _____
2. The representation of complex number $z = 2i$ into exponential form is _____
3. The domain of the function $f(z) = \frac{1}{1-|z|^2}$ is _____
4. A twice differential function $u = u(x, y)$ of two variables is said to be harmonic if _____
5. The value of principal logarithm $\text{Log}(-1) =$_____
6. The value of the integral is $\int_0^{i/2} \pi e^{\pi z}\, dz =$ _____
7. For any analytic function $f(z)$ on and inside a simples close contour C, the value of the integral $\oint_C f(z)dz =$ _____
8. The sequence of the complex numbers $z_n = 1 + \frac{n+1}{n} i$ converges to the limit _____ as $n \to \infty$.
9. The singular points of the function $f(z) = \frac{z^2+1}{z(z-i)^3}$ are _____ and _____
10. The image of linear fractional transformation $f(z) = \frac{az+b}{cz+d}, (ad - bc \neq 0)$ on the extended complex plane at $z = \infty$ is _____

SECTION "B"
[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s), ***DO NOT TICK,*** *by selecting the most appropriate answers from among the given ones.*

11. If a complex number z is equal to its complex conjugate $\bar{z}$, then the complex number must be _____
[pure imaginary; real; undefined; ∞]

12. The principal argument of the complex number $z = -3i$ is _____
[$-\frac{\pi}{2}$; 0; $\frac{\pi}{2}$; π]

13. If a complex function $f(z) = u(x, y) + iv(x, y)$ is analytic in some domain D, then its derivative can be calculated by the formula $f'(z) =$ _____ $\forall\, z \text{ in } D$.
[$u_x + iv_y$; $v_x + iu_x$; $v_y + iu_y$; $u_x + iv_x$]

14. The complex exponential function $f(z) = e^z$ is the periodic function of period _____
[$\frac{\pi}{2}i$; πi; $2\pi i$; 2π]

15. The value of the complex integral $\oint_C \frac{1}{z-z_0} dz =$ _____, where C is a circle: $|z - z_0| = R$ oriented positively.
[0; 1; 2π; $2\pi i$]

16. The region defined by $1 < |z| < 2$ represents the _____
[open disk; circle;
simply connected domain; doubly connected domain]

17. The series $\sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}$ for all $|z| < \infty$ converges to the function $f(z) =$_____
[$\sin z$; $\sinh z$; $\cosh z$; $\cos z$]

18. The function $f(z) = \tan\frac{1}{z}$ has _____ singularity at $z = 0$.
[an essential; a removal; an isolated; a non-isolated]

19. The residue of $f(z) = \frac{1}{z(i-z)}$ at the singular points $z = i$ is _____
[-1; $-i$; i; 0]

20. The mapping $f(z) = -iz$ rotates every complex number z through the angle of_____ about origin.
[$-\frac{\pi}{2}$; $-\pi$; π; $\frac{\pi}{2}$]

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Level : B.E.
Year : III
Time : 2 hrs. 30mins.

JUL 2024

Course : MATH 326
Semester : I
F. M. : 50

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define the continuity and differentiability of a complex valued function $f(z)$ at $z = z_0$. Prove that if $f(z)$ is differentiable at a point $z = z_0$, then it is continuous at $z = z_0$. Show that the function [1+2+4]

$f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & when\ z \neq 0 \\ 0, & when\ z = 0 \end{cases}$ is not differentiable at $z = 0$ but continuous at $z = 0$.

OR

Define a harmonic function. Show that if a complex variable function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D, then its component functions $u = u(x,y)$ and $v = v(x,y)$ both are harmonic in D. Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2$ is harmonic and find its harmonic conjugate $v = v(x,y)$. [1+3+3]

2. State and prove that the Cauchy-integral formula. Using Cauchy-integral formula evaluate the integral $\int_C \frac{e^{\pi z}}{z^2+4} dz$, where C is the circle $|z + i| = 2$ in the positive sense. [4+3]

3. State and prove the residue theorem for the integration of an analytic function $f(z)$ on a closed contour C that enclosed finite number of singular points of $f(z)$. Using this theorem, evaluate the integral $\oint_C \frac{(3z+2)^2}{z(z-1)^2(2z+5)} dz$, where C be the circle $|z| = 2$ oriented counter-clockwise. [4+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Find all the roots of $(1 - i)^{1/3}$ in rectangular coordinates and point out which is the principal root.

5. Show that $\sin z = \sin x \cosh y + i \cos x \sinh y$, where $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$ is complex Sine function.

OR

Write $|e^{2z+i}|$ and $|e^{iz^2}|$ in terms of x and y. Then, show that $|e^{2z+i} + e^{iz^2}| \leq e^{2x} + e^{-2xy}$.

6. For any two integers m and n, evaluate the integral $\int_C z^m \bar{z}^n dz$, where C is the circle $|z| = 1$ taken counter-clockwise.

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7. Develop the Laurent series of the function $f(z) = \frac{e^z}{(z+1)^2}$ at the singular point. Write the principal part of $f(z)$, classify the type of singularity.

8. Using complex integration evaluate the integral $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

9. Find the linear fractional transformation that maps the points $z_1 = -1$, $z_2 = 0$, $z_3 = 1$ onto the points $w_1 = -i$, $w_2 = 1$, $w_3 = i$ respectively.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Prove that $\sqrt{2}|z| \geq |Rez| + |Imz|$.

11. Write the function $f(z) = z + \frac{1}{z}, z \neq 0$ into the polar form $f(z) = u(r,\theta) + iv(r,\theta)$.

12. Find all values of z such that $e^z = 1 + \sqrt{3}i$.

13. Find the residue of the function $f(z) = \frac{z^3+1}{(1+z)^2}$ at the singularity.

14. Write the Taylor's series of the function $f(z) = e^z$ at $z = -i$.