KATHMANDU UNIVERSITY
End Semester Examination
January/February 2024

Level : B.Sc. | Course : MATH 326
Year : III | Semester : I
Time : 2 hrs. 30mins. | F. M. : 55

22 JAN 2024

SECTION "C"
[3 Q. × 7 = 21 marks]

1. State Cauchy-Gaursat theorem and explain how it differs with Cauchy integral theorem. Show that if the function $f(z)$ is analytic, then the value of the integral $\int_C f(z)dz$ is independent of the path C joining two points. Evaluate the integral $\oint_C \frac{e^{-z}}{z-i\frac{\pi}{2}}dz$, where C denotes the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$, where C is described in the positive sense. [1 + 1 + 3 + 2]

OR

Prove that the Cauchy-integral formula for derivative $f'(z_0) = \frac{1!}{2\pi i}\oint_C \frac{f(z)}{(z-z_0)^2}dz$, where the function $f(z)$ which is analytic within and on a positively oriented simple close contour C that encloses the point z_0. Find the value of the integral $\oint_C \frac{1}{(z^2+4)^2}dz$, where C is the circle $|z - i| = 2$ oriented positively. [4+3]

2. State and prove the Taylor's theorem for the series representation of an analytic function $f(z)$. Use this theorem to express series expansion of the function $f(z) = \sin z$ at $z = 0$. [1 + 4 + 2]

3. Define the Cauchy principal value (P.V.) of an improper integral $\int_{-\infty}^{\infty} f(x)dx$. State the conditions for the real valued function $f(x)$ to evaluate such integrals using complex integration theory. Use complex integration to evaluate the integral $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}dx$. [1+2+4]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Establish the identity $1 + z + z^2 + \cdots + z^n = \frac{1-z^{n+1}}{1-z}$ for any complex number z and use it to show that $1 + \omega + \omega^2 + \cdots + \omega^{n-1} = 0$, where ω denotes an nth root of unity other than 1.

5. Show that the function $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & when\ z \neq 0 \\ 0, & when\ z = 0 \end{cases}$ is continuous at $z = 0$.

6. Show that the function $u = y^3 - 3x^2y$ is harmonic and find its conjugate harmonic $v = v(x, y)$.

OR

Show that the function $f(z) = e^{-\theta}\cos(ln r) + ie^{-\theta}\sin(ln r), r > 0, 0 < \theta < 2\pi$ is differentiable in the indicated domain of definition, and also show that its derivative $f'(z) = i\frac{f(z)}{z}$.

7. Find the value of the integral $\oint_C (z - z_0)^m dz$, where C is $z(t) = z_0 + Re^{it}, 0 \le t \le 2\pi$.

8. Develop the Laurent series of the function $f(z) = ze^{1/z}$ at the singular point. Write the principal part of $f(z)$, classify the type of singularity and also find the residue at singular point.

9. Find the linear fractional transformation that maps the points $z_1 = -i$, $z_2 = 0$, $z_3 = i$ onto the points $w_1 = -1$, $w_2 = i$, $w_3 = 1$ respectively.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find square roots of the complex number $z = i$.

11. Sketch the region for set of complex numbers z such that $0 \leq arg\, z \leq \pi/4$ and explain whether it is closed and bounded.

12. Find all possible values of the complex exponent i^i.

13. Evaluate the complex integral $\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$.

14. Write the Taylor's series of the function $f(z) = 1/z$ at $z = i$.

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Registration No.:

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SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The complex conjugate of the complex number $z = 3 - 2i$ is $\bar{z} =$ ....

2. The value of $i^8 =$ ...., where i denotes the imaginary unit.

3. The modulus of the complex number $z = 2e^{i\theta}$ is ....

4. The value of the function $f(z) = 2xy + i(x^2 + y^2)$ at the point $z = -i$ is ....

5. A differential function $f(z) = u + iv$ satisfies following two Cauchy-Riemann equations .... and ....

6. The value of principal logarithm $\text{Log}\, i =$............................... .

7. The value of the complex integral $\oint_C e^z dz =$ ...., where C is unit circle oriented positively.

8. The sequence of the complex numbers $z_n = 2 + \frac{i(-1)^n}{n^2}$ converges to $z =$ .... as $n \to \infty$.

9. The function $f(z) = \frac{e^z}{z(z-i)^3}$ has a pole of order .... at $z = i$.

10. The mapping $f(z) = (1 + i)z$ rotates every complex number z through the angle of .... about origin.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The principal argument of the complex number $z = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ is ....
[1; $-\frac{\pi}{4}$; $\frac{\pi}{4}$; $\frac{3\pi}{4}$]

12. If $\bar{z}$ denotes complex conjugate of a complex number z, then $z + \bar{z} =$.............................. .
[$2Rez$; $2Imz$; $2iRez$; $2iImz$]

13. A cube root of unity is................................ .

[-1; $e^{i\frac{\pi}{3}}$; $e^{i\frac{2\pi}{3}}$; $e^{i\frac{3\pi}{3}}$]

14. The unit disk $|z| \leq 1$ is ....

[open and bounded; closed and unbounded; doubly connected; closed and bounded]

15. The Taylor's series $\sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$ for all $|z| < \infty$ converges to the function $f(z) =$........................ .

[$\sin z$; $\sinh z$; $\cosh z$; $\cos z$]

16. The function $f(z) = \frac{\sin z}{z}$ has .... singularity at $z = 0$.

[an essential; a removal; no; a non-isolated]

17. The parametric representation of line segment joining from $z = 0$ to $z = 1 + i$ is ...., $0 \leq t \leq 1$.

[$z(t) = -t - it$; $z(t) = t + it$; $z(t) = t - it$; $z(t) = -t + it$]

18. $\lim_{z\to\infty} f(z) = w_0$ if and only if ....

[$\lim_{z\to 0} \frac{1}{f(z)} = w_0$; $\lim_{z\to 0} \frac{1}{f\left(\frac{1}{z}\right)} = w_0$; $\lim_{z\to 0} f\left(\frac{1}{z}\right) = w_0$; $\lim_{z\to 0} f(z) = 0$]

19. The residue of $f(z) = \frac{z}{z^2+1}$ at the singular points $z = \pm i$ is ....

[$\frac{1}{2}$; $-\frac{1}{2}$; $\frac{i}{2}$; $\frac{i}{2}$]

20. The inversion mapping $w = \frac{1}{z}$ maps a circle through the origin in z-plane into .... in the w-plane.

[circle through origin; circle not through origin; line through origin; line not through origin]