KATHMANDU UNIVERSITY
End Semester Examination [C]
December, 2024

	Marks Scored:
Level : B.E.	Course : MATH 326
Year : III	Semester : I
Exam Roll No. : Time: 30 mins.	F. M. : 20
Registration No.:	Date 20 DEC 2024

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. The distance measured of the complex point $z = 1 + i$ from origin is ………………units.

2. The principal argument of the complex number $z = 1 - i$ is ………………………. .

3. If $f(z) = \frac{2z-3}{-z+1}$, then the limit $\lim_{z\to\infty} f(z) =$ ……………………. .

4. A function $u = u(x, y)$ is harmonic in a given domain if it satisfies ……………….. .

5. The value of principal logarithm $\text{Log}(-i) =$……………………………. .

6. The length of the curve given by $z(t) = e^{it}, 0 \le t \le 2\pi$ is ………………………. .

7. The value of the complex integral $\int_0^i z\,dz$ ……………………. .

8. The sequence of the complex numbers $z_n = -1 + \frac{i^n}{n^2}$ converges to $z =$ ……………… as $n \to \infty$.

9. The function $f(z) = \frac{\sinh z}{z^4}$ has Laurent series expansion $f(z) = \frac{1}{z^3} + \frac{1}{3!}\frac{1}{z} + \frac{1}{5!}z + \frac{1}{7!}z^3 + \cdots$ at the pole $z = 0$. Then the residue of $f(z)$ at $z = 0$ is ………………………… .

10. The mapping $f(z) = iz$ rotates every complex number z through the angle of ……………… about origin.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), ***DO NOT TICK,*** *by selecting the most appropriate answers from among the given ones.*

11. If $z = i$, then its multiplicative inverse z^{-1} is ______________________.
    [1; i; $-i$; -1]

12. The value of i^{20} = ______________, where i denotes the imaginary unit.
    [-1; $-i$; i; 1]

13. If a complex function $f(z)$ is analytic in a domain D, then it must satisfy the ______________ equation(s).
    [Cauchy-Euler; Cauchy-Goursat; Cauchy-Riemann; Cauchy-intergral]

14. If a complex function $f(z) = u(x,y) + i\, v(x,y)$ analytic in a domain D, then $v = v(x,y)$ is ______________ of $u = u(x,y)$.
    [harmonic conjugate; inverse; harmonic; conjugate]

15. The complex exponential function $f(z) = e^z$ is a periodic function of period ______________.
    [π; πi; 2π; $2\pi i$]

16. The parametric representation of the unit circle $|z| = 1$ oriented positively is given by ______________, $0 \le t \le 2\pi$.
    [$z(t) = e^t$; $z(t) = e^{-t}$; $z(t) = e^{it}$; $z(t) = e^{-it}$]

17. The Taylor's series $\sum_{n=0}^{\infty}(-1)^n z^n$ for all $|z| < 1$ converges to the function $f(z) =$ ______________.
    [$\frac{1}{1+z}$; $\frac{1}{1-z}$; $\cos z$; $\sin z$]

18. The function $f(z) = \frac{e^z}{z(z-i)^3}$ has simple pole at $z =$ ______________.
    [0; 1; i; -1]

19. The function $f(z) = e^{\frac{1}{z}}$ has ______________ singularity at $z = 0$.
    [an essential; a removal; no; a non-isolated]

20. The residue of $f(z) = \frac{4z}{z^2+1}$ at the singular point $z = i$ is ______________.
    [0; 2; 1; -2]

KATHMANDU UNIVERSITY
End Semester Examination [C]
December, 2024

20 DEC 2024

Level : B.E.
Year : III
Time : 2 hrs. 30mins.

Course : MATH 326
Semester : I
F. M. : 50

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Derive the Cauchy-Riemann equations for a differential function $f(z) = u(x, y) + iv(x, y)$ at a point $z_0 = x_0 + iy_0$. Show that the function $f(z) = (x^2 - y^2) + i2xy$ is differentiable for all $z = x + iy$ and also find its derivative $f'(z)$. [4+3]

OR

Define the derivative of a complex variable function $w = f(z)$ at a point $z = z_0$. Show that the function $f(z) = \begin{cases} \frac{\bar{z}^2}{z} & when\ z \neq 0 \\ 0, & when\ z = 0 \end{cases}$ satisfies the Cauchy-Riemann equations at origin $z = 0$ but has no derivative at $z = 0$. [1+3+3]

2. State and prove that the Cauchy-integral formula for a complex variable function $f(z)$. Using Cauchy-integral formula, evaluate the integral $\oint_C \frac{e^z}{z-2i} dz$, where C is the contour $|z - 2i| = 4$ in the positive sense. [1+4+2]

3. State and prove the residue theorem for the integration of an analytic function $f(z)$. Find the value of the integral $\oint_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$, where C is the contour $|z| = 4$ in the positive sense. [4+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Prove the triangle inequality $|z_1 + z_2| \leq |z_1| + |z_2|$ for any two complex numbers z_1 and z_2.

5. Does the limit of the function $f(z) = \left(\frac{z}{\bar{z}}\right)^2$ exist at $= 0$? Explain with reasons.

6. Verify that the function $u = x^2 - y^2 - y$ is harmonic in whole complex plane and find a harmonic conjugate $v = v(x, y)$ of u.

P.T.O.

7. Find the value of the integral $\oint_C (z + \bar{z})\, dz$, where C is the circle $|z| = 1$ oriented counter-clockwise.

8. Find the Cauchy-Principal value of the integral $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2}\, dx$.

OR

Using complex integration to show that $\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta} = 2\pi$.

9. Find the linear fractional transformation that maps the points $z_1 = 0, z_2 = -i, z_3 = i$ onto the points $w_1 = -1, w_2 = 0, w_3 = \infty$ respectively.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Sketch the region for set of complex numbers z satisfying the condition $|z + i| \leq 3$ and explain whether it is closed and bounded.

11. Write the function $f(z) = z^2 + \bar{z}^2$ into the form $f(z) = u(x, y) + v(x, y)$.

12. Find all possible values z which satisfies the equation $e^z = 1 + i$.

13. Find all the singular points of the function $f(z) = \frac{1}{\sin \pi z}$ and classify the isolated and non-isolated singularities.

14. Write the Laurent series of the function $f(z) = \frac{e^z}{z^2}$ at the singular point $z = 0$. .