KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc.
Year : III

Course : MATH 325
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 10

Registration No.:

Date 03 JUL 2023

SECTION "A"
[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The infimum of the set (2, 10) is ....

2. A point in n-dimensional space is an ....of real numbers.

3. The union of a finite collection of closed sets is ....

4. A set M in $\Re^n$ is said to be compact if and only if, every open covering of M contains a ....

5. Let S be a subset of a metric space M. A point x in M is called a boundary point of S if every ball $B_M(x, r)$ contains at least one point of S and at least one point of ....

6. In any metric space (M, d) every .... subset T is complete.

7. In Euclidean space $\Re^1$, a sequence $\{x_n\}$ is called decreasing if .... for all n.

8. In Euclidean space $\Re^k$ every .... sequence is convergent.

9. A partition P of $[a, b]$ is said to be finer than P' (or a refinement of P') if ....

10. If f is ....on $[a, b]$, then f is of bounded variation on $[a, b]$.

SECTION "B"
[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. **(DO NOT TICK THE ANSWER)**.

11. The symmetric property of metric d is...............................
[$d(x,y) = d(y,x)$; $d(x,y) = d(x,y)$; $(x,y) \geq d(y,x)$; $d(x,y) \leq d(x,z) + d(z,y)$]

12. If (M, d) is a metric space and $a \in M$, the ball $B(a,r)$ with center a and radius $r > 0$ is defined to be the set of all x in M such that ....
[$d(x,a) > r$.; $d(x,a) \leq r$.; $d(x,a) < r$.; $d(x,a) \geq r$.]

13. A convergent sequence has only .... limit(s).
[one; two; three; less than three]

14. A function $f : S \to \Re^k$ is called bounded on S if there is a positive number M such that $f(x)$................ M for all x in S.
[$<$; $>$; $\leq$; $\geq$]

15. A metric space (S, d) is called complete if every Cauchy sequence in S .... in S.
[converges; diverges; continuous; discontinuous]

16. Assume f and g are defined on (a, b) and differentiable at c. Then $(gf)'(c) =$ ....
[$f(c)g'(c) + f(c)g(c)$; $f(c)g'(c) + f'(c)g(c)$; $f(c)g(c) + f'(c)g(c)$; $f(c)g(c) + f(c)g'(c)$]

17. If the function $f : \Re \to \Re$ be defined by $f(x) = 2x - 7$, then $fof^{-1}(100) =$ ....
[50; 100; 200; 300]

18. If f is differentiable on $[a, b]$ then it is monotonically decreasing if.............
[$f(x) = 0$; $f'(x) = 0$; $f'(x) \geq 0$; $f'(x) \leq 0$]

19. If f is of bounded variation on $[a, b]$ then on $[a, b]$, f is..........
[unbound; bounded; continuous; zero]

20. $\int_a^b f d\alpha + \int_b^c f d\alpha + \int_c^a f d\alpha =$ ....
[f; 0; α; a]

KATHMANDU UNIVERSITY
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03 JUL 2023

Level : B.Sc.
Year : III
Time : 2 hrs. 30 mins.

Course : MATH 325
Semester : II
F. M. : 50

SECTION "C"
[6Q. × 7 = 42 marks]

1. State and prove triangle inequality in Euclidean space R^n. Prove that the union of any collection of open sets is an open set [1+3+3]

2. Define complete metric space. If $M = \mathfrak{R}^n$ and $d = \|\boldsymbol{x} - \boldsymbol{y}\|$, then prove that (M, d) is a metric space. In any metric space (S, d), prove that every compact subset T is complete. [1+3+3]

OR

Define bounded and compact set. Let $f : S \to T$ be a function from one metric space (S, d_S) to another (T, d_T). Then f is continuous on S if and only if for open set Y in T, the inverse image $f^{-1}(Y)$ is open in S. [2+5]

3. Define derivative and partial derivative. Let f be defined on an open interval S, let g be defined on $f(S)$ and consider the composite function gof defined on S by the equation $(gof)(x) = g[f(x)]$. Assume there is a point c in S such that $f(c)$ is an interior point of $f(S)$. If f is differentiable at c and if g is differentiable at $f(c)$ then gof is differentiable at c, then prove that $(gof)'(c) = g'[f(c)]f'(c)$. [2+5]

4. Define the bounded variation and total variation. Let f be of bounded variation on $[a, b]$. Let V be defined on $[a, b]$ as follows: $V(x) = V_f(a, x)$, if $a < x \leq b$, $V(a) = 0$. Then prove that [2+2+3]
   a. V is an increasing function on $[a, b]$.
   b. $V - f$ is an increasing function on $[a, b]$.

5. What is finer partition of partition P of $[a, b]$? Define Riemann-Stieltjes sum of f with respect to α. If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on $[a, b]$, then $(c_1 f + c_2 g) \in R(\alpha)$ on $[a, b]$ and c_1, c_2 are any two constants then prove that: $\int_a^b (c_1 f + c_2 g) d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$ [2+5]

OR

Define partition of $[a, b]$ and Riemann-Stieltjes integral function. Assume $f \in R(\alpha)$ on $[a, b]$ and assume that α has a continuous derivative α' on $[a, b]$. Then the Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists then prove that:

$$\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$$ [2+5]

6. Define uniform convergence sequence and uniform convergence series of a functions. Let $\{f_n\}$ be a sequence of functions defined on a set S. There exists a function f such that $f_n \to f$ uniformly on S if, and only if, the following condition is satisfied: For every $\varepsilon > 0$ there exists an N such that $m > N$ and $n > N$ implies $|f_m(x) - f_n(x)| < \varepsilon$, for every x in S. [2+5]

SECTION "D"
[4Q. × 2 = 8 marks]

7. If f is differentiable at a point x_0, then prove that f is continuous at x_0.

8. If x is an accumulation point of S, then prove that every n-ball $B(x)$ contains infinitely many points of S.

9. Let $f : S \to T$ be a function from S to T. If $X \subset S$ and $Y \subset T$, then prove that $Y = f(X)$ implies $X \subset f^{-1}(Y)$.

10. If f and g are defined on (a, b) and differentiable at c. Then prove that
$(f + g)'(c) = f'(c) + g'(c)$.