KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

Marks Scored:

Level : B.E.
Year : III

Course : MATH 325
Semester : II
F. M. : 10

Exam Roll No. :

Time: 30 mins.

Registration No.:

Date 05 AUG 2024

SECTION "A"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. A point in $\Re^n$ space is an ____________________ of real numbers.

2. The suprimum of the set (0, 20] is ________________

3. The intersection of any collection of closed sets is ____________________

4. A set M in $\Re^n$ is said to be ________________ if and only if, every open covering of M contains a finite sub-cover,

5. Let S be a subset of a metric space M. A point x in M is called a __________________ point of S if every ball $B_M(x, r)$ contains at least one point of S and at least one point of M – S.

6. In any metric space (M, d) every ____________________ subset T is complete.

7. In Euclidean space R^1, a sequence $\{x_n\}$ is called increasing if ____________________ for all n

8. In Euclidean space $\Re^k$ every Cauchy sequence is __________________ sequence

9. A partition P of [a, b] is said to be finer than partition P' (or a refinement of P') if ____________________

10. If f is monotonic on [a, b], then f is of ___________ variation on [a, b].

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (**DO NOT TICK** THE ANSWER).

11. The triangle inequity property of metric d is ____________________

[$d(x, y) = d(y, x)$; $d(x, y) = 0$; $d(x, y) \geq 0$; $d(x, y) \leq d(x, z) + d(z, y)$]

12. If (M, d) is a metric space and $a \in M$, the ball B(a, r) with center a and radius $r > 0$ is defined to be the set of all x in M such that ____________________

[d (x, a) > r.; d (x, a) $\leq$ r.; d (x, a) < r.; d (x, a) $\geq$ r.]

13. A ______________ sequence has only one limi.
[uncountable; countable; convergent; divergent]

14. A function $f : S \rightarrow \Re^k$ is called bounded on S if there is a positive number M such that _______ M for all x in S.

[$\|f(x)\| <$; $\|f(x)\| >$; $\|f(x)\| \leq$; $\|f(x)\| \geq$]

15. A metric space (S, d) is called ______________ if every Cauchy sequence in S converges in S.

[unbound. bounded complete. finite]

16. Assume f and g are defined on (a, b) and differentiable at c. Then (g f)'(c) = ____________________

[f(c)g'(c) + f (c)g(c) ; f(c)g'(c) + f '(c)g(c) ;

f(c)g(c) + f '(c)g(c) ; f(c)g(c) + f (c)g' (c)]

17. If the function $f : \Re \rightarrow \Re$ be defined by $f(x) = x^2 - 7$, then fof $^{-1}(300)$ = __________________________

[50; 100; 200; 300]

18. If f is differentiable on [a, b] then it has local extreme value(s) at x = a if __________________________

[f(a) = 0; f'(a) = 0; f'(a) $\geq$ 0; f'(a) $\leq$ 0]

19. If f is of bounded variation on [a, b] then on [a, b] f is____________________

[unbound; non zero; continuous; zero]

20. $\int_a^b f d\alpha + \int_b^c f d\alpha + \int_c^a f d\alpha =$ __________________________
[a + b + c; 0; α; a]

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05 AUG 2024

SECTION "C"
[6 Q. × 7 = 42 marks]

1. Define adherent point of a set. Prove that the union of any collection of open sets is an open set and union of finite collection of closed sets is closed. [1+3+3]

2. Define open covering and compact set on metric space (M, d). Let f : S → T be a function from one metric space (S, d_S) to another (T, d_T). Then f is continuous on S if and only if for open set Y in T, the inverse image $f^{-1}(Y)$ is open in S. [2+5]

OR

Define metric space and complete metric space. Let (M, d) is a metric space, define $d^1 = \frac{d(x,y)}{1+d(x,y)}$ for all x, y in M, and then prove that (M, d^1) is a metric space. [1+1+5]

3. Define left hand derivative and partial derivative. If f is differentiable at a point x_0, then prove that f is continuous at x_0 but converse may not be true. [1+1+2+3]

4. Define the partition and bounded variation. Let f be of bounded variation on [a, b]. Let V be defined on [a, b] as follows: $V(x) = V_f(a, x)$, if $a < x \le b$, $V(a) = 0$. Then prove that
   a. V is an increasing function on [a, b].
   b. V-f is an increasing function on [a, b]. [1+1+2+3]

5. What is finer partition of partition P of [a, b]?. If f ∈ R(α) and if g ∈ R(α) on [a, b], then (c_1 f + c_2g) ∈ R(α) on [a, b] and c_1, c_2 are any two constants then prove that: $\int_a^b f d(c_1\alpha + c_2\beta) = c_1 \int_a^b f d\alpha + c_2 \int_a^b f d\beta$ [2+5]

OR

Define Riemann-Stieltjes sum of f with respect to α and Riemann-Stieltjes integral function. Assume f ∈ R(α) on [a, b] and assume that α has a continuous derivative α ' on [a, b]. Then the Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists then prove that:
$\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$ [2+5]

6. Define uniform convergence of sequences of functions. State and prove that the Cauchy condition for uniform convergence of sequences of functions. [1+1+5]

P.T.O.

SECTION "D"

[4 Q. × 2 = 8 marks]

7. Let x and y denote points in $\Re^n$. Then prove that: $\|x + y\| \leq \|x\| + \|y\|$.

8. Let $f : S \to T$ be a function from S to T. If $X \subset S$ and $Y \subset T$, then prove that $Y = f(X)$ implies $X \subset f^{-1}(Y)$.

9. Prove that in Euclidean space R^k every Cauchy sequence is convergent.

10. If f and g are defined on (a, b) and differentiable at c. Then prove that $(f - g)'(c) = f'(c) - g'(c)$.