KATHMANDU UNIVERSITY
End Semester Examination
February, 2025

Marks Scored:

Level : B.Sc.
Year : III

Course : MATH 325
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 10

Registration No.:

Date 18 FEB 2025

SECTION "A"

[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. The unit coordinate vector $\mathbf{u}_k$ in $\Re^n$ is the vector whose k^{th} component is ____________________ and whose remaining components are 0.

2. A point in 2- dimensional space is a/an ____________________ of real numbers.

3. The intersection of a finite collection of closed sets is ____________________

4. A set M in $\Re^n$ is said to be compact if and only if, every open covering of M contains a ____________________

5. Let S be a subset of a metric space M. A point x in M is called a boundary point of S if every ball $B_M(x, r)$ contains at least one point of ____________________ and at least one point of $M - S$.

6. If A is open and B is closed in metric space (M, d), then A - B is ____________________.

7. In Euclidean space $\Re^1$, a sequence $\{x_n\}$ is called increasing if ____________________ for all n

8. In Euclidean space $\Re^k$ every Cauchy sequence is ____________________.

9. A partition P' of [a, b] is said to be finer than P (or a refinement of P) if ____________________

10. If f is defined on [a, b]. If $P = \{x_0, x_1, \dots, x_n\}$ is a partition of [a, b], If there exists a positive number M such that $\sum_1^n |\Delta f_k|$ ____________________ for all partitions of [a, b], then f is said to be bounded variation on [a, b].

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. ***(DO NOT TICK THE ANSWER)***

11. The triangle inequality property of metric d is ____________
[d(x, y) = d(y, x); d(x, y) = d(x,y) ;
(x, y) ≥ d(y, x); d(x, y) ≤ d(x, z) + d(z, y)]

12. If (M, d) is a metric space and a ∈ M, the ball B(a, r) with center a and radius r > 0 is defined to be the set of all x in M such that ____________
[d (x, a) > r.; d (x, a) ≤ r.; d (x, a) < r.; d (x, a) ≥ r.]

13. A metric space (S, d) is called complete if every Cauchy sequence in S ________
[Converges in S; diverge in S; converge; bounded on S]

14. A function f : S → $\Re^k$ is called bounded on S if there is a positive number M such that f(x)____________M for all x in S.
[< ; > ; ≤; ≥]

15. If f is continuous at every point of a subset A of S, we say f is. ____________.
[continuous; everywhere continuous;
continuous on S; continuous on A]

16. Assume f and g are defined on (a, b) and differentiable at c. Then (g f)'(c) = ____________
[f(c)g'(c) + f (c)g(c) ; f(c)g'(c) + f '(c)g(c);
f' (c) + g' (c); f' (c) - g' (c)]

17. If the function f : $\Re \rightarrow \Re$ be defined by f(x) = $2x^2 - 7$, then fof $^{-1}$(10) = ____________
[5; 10; 20; 30]

18. If f is differentiable on [a, b] then it is monotonically decreasing if____________
[f(x) = 0; f'(x) = 0; f'(x) ≥ 0; f'(x) ≤ 0]

19. If f is of bounded variation on [a, b] then on [a, b] f is____________
[unbound; bounded; continuous; zero]

20. $\int_a^b f d\alpha + \int_b^a f d\alpha$ = ____________
[f; 0; α; a]

KATHMANDU UNIVERSITY
End Semester Examination
February, 2025

Level : B.Sc.
Year : III
Time : 2 hrs. 30mins.

18 FEB 2025

Course : MATH 325
Semester : II
F. M. : 50

SECTION "C"
[6Q. × 7 = 42 marks]

1. Define interior point and open set in Euclidean space $\Re^n$. Prove that the union of any collection of open sets is an open set and intersection of any collection of closed set is a closed. [2+3+2]

2. Define metric space. Let (M, d) is a metric space, define $d^1 = \frac{d(x,y)}{1+d(x,y)}$ for all x, y in M, and then prove that (M, d^1) is a metric space. In any metric space (S, d), prove that every compact subset T is complete. [1+4+2]

OR

Let f : S → T be a function from one metric space (S, d_S) to another (T, d_T). Then f is continuous on S if and only if for open set Y in T, the inverse image $f^{-1}(Y)$ is open in S. Also prove that in Euclidean space R^k every Cauchy sequence is convergent. [4+3]

3. Define derivative and right hand derivative. Let f be defined on an open interval S, let g be defined on f(S) and consider the composite function gof defined on S by the equation (gof)(x) = g[f(x)]. Assume there is a point c in S such that f(c) is an interior point of f(S). If f is differentiable at c and if g is differentiable at f(c) then gof is differentiable at c, then prove that (gof)'(c) = g'[f(c)]f '(c). [2+5]

4. Define Riemann-Stieltjes sum and integral of f with respect to α. If f ∈ R(α) and if g ∈ R(α) on [a, b], then (c_1 f - c_2g) ∈ R(α) on [a, b] and c_1, c_2 are any two constants then prove that: $\int_a^b (c_1 f - c_2 g)d\alpha = c_1 \int_a^b f d\alpha - c_2 \int_a^b g d\alpha$ [2+5]

OR

Define partition of [a, b] and finer partition. Assume f ∈ R(α) on [a, b] and assume that α has a continuous derivative α ' on [a, b]. Then the Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists then prove that:

$\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$ [2+5]

5. Define uniform convergence sequence and uniform convergence series of functions. State and prove the Cauchy condition for uniform convergence of series of functions. [2+1+4]

P.T.O.

6. Define the partition, bounded variation and total variation. Let f be of bounded variation on [a, b]. Let V be defined on [a, b] as follows: $V(x) = V_f(a, x)$, if $a < x \leq b$, $V(a) = 0$. Then prove that

a. V is an increasing function on [a, b].

b. V-f is an increasing function on [a, b]. [3+2+2]

SECTION "D"

[4Q. × 2 = 8 marks]

7. Prove that every continuous function may not be differentiable.

8. Let $f : S \rightarrow T$ be a function from S to T. If $X \subset S$ and $Y \subset T$, then prove that $Y = f(X)$ implies $X \subset f^{-1}(Y)$.

9. If f and g are defined on (a, b) and differentiable at c. Then prove that $(f - g)'(c) = f'(c) - g'(c)$.

10. Use Riemann-Stieltjes integral. Prove that $\int_a^b df(x) = f(b) - f(a)$,