KATHMANDU UNIVERSITY

End Semester Examination

May/June, 2022

Level : B.Sc.
Year : III
Exam Roll No. :

Time: 30 mins.

Course : MATH 322
Semester : II
F. M. : 20

Registration No.:

Date :

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The dual statement of the statement $\neg(p \vee q)$ is ______________________, where p and q are primitive statements.

2. The number of 3-combinations of $\{\infty \cdot a,\ 2 \cdot b, 1 \cdot c\}$ is ______________________.

3. Let $a, b \in \mathbb{Z}^+$ and $d = GCD(a, b)$. Then $GCD(-a, -b) =$ ______________________.

4. Consider the integer sequence $a_0, a_1, a_2, \cdots$ where $a_0 = 1,\ a_1 = 2,\ a_2 = 3$ and $a_{n+3} = a_n + a_{n+1} + a_{n+2},\ n \geq 0$. Then $a_5 =$ ______________________.

5. The number of non-negative integral solutions to $e_1 + e_2 + \cdots + e_{10} = 8$ where each $e_i \geq 0$ is ______________________.

6. If G is a finite group and H be a subgroup of G. Then the ______________________ theorem states that $0(H)$ divides $0(G)$.

7. The generating function $\frac{1}{(1-x)^2}$ generates the sequence ______________________.

8. Let G be a group and $a, b \in G$. Then $(ab)^{-1} =$ ______________________.

9. The total number of positive divisor of 360 is ______________________.

10. Let A, B be subsets of a universal set Ω. Then the absorption law gives $A \cup (A \cap B) =$ ______________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK THE ANSWER**, by selecting the most appropriate answer from among the given ones.

11. The coefficient a_r of x^r in the expansion of $\frac{1}{(1-x)^3}$ is ______________________.

[r ; $r + 1$; $\frac{r+1}{2}$; $\frac{(r+1)(r+2)}{2}$]

12. The function $\phi : \mathbb{Z}_6 \to \mathbb{Z}_6$ defined by $\phi(x) = 3x$ is a group homomorphism under integer modulo 6. Then the kernel of ϕ, ker ϕ = ____________________.
[{0}; {2}; {0, 2}; {0, 2, 4}]

13. If $P(n), n \in \mathbb{Z}^+$ denotes the number of partitions of n into positive summands. Then $P(n)$ is the coefficient of x^n in ____________________.
[$\sum_{i=1}^{n} \frac{1}{1-x^i}$; $\prod_{i=1}^{n} \frac{1}{1-x^i}$; $\sum_{i=1}^{n} 1 - x^i$; $\prod_{i=1}^{n} 1 - x^i$]

14. Let H be a subgroup of a finite group G. Then the number of distinct left (right) cosets of H in G is equal to ____________________.
[$0(H)$; $0(G)$; $\frac{0(G)}{0(H)}$; $\frac{0(H)}{O(G)}$]

15. Let $a, b \in \mathbb{Z}$ and $d = GCD(a, b)$. Then ____________________ identity states that there exists integers x and y such that $d = ax + by$.
[Fermat; Pell; Bezout; Diophantine]

16. The negation of the statement $(\forall\, x)(\exists\, y)[p(x,y) \wedge q(x,y) \Rightarrow r(x,y)]$ is ____________________.
[$(\forall\, x)(\exists\, y)[\neg(p(x,y) \vee q(x,y)) \vee \neg r(x,y)]$;
$(\exists\, x)(\forall\, y)[\neg(p(x,y) \wedge q(x,y)) \vee r(x,y)]$;
$(\forall\, x)(\exists\, y)[(p(x,y) \vee q(x,y)) \wedge \neg r(x,y)]$;
$(\exists\, x)(\forall\, y)[(p(x,y) \wedge q(x,y)) \vee \neg r(x,y)]$]

17. A derangement D_n is a permutation of n distinct items $a_1, a_2, \cdots, a_n$ such that no item a_i occupies position i, $1 \le i \le n$. Then D_3 = ____________.
[0; 1; 2; 9]

18. Every nonempty subset of $\mathbb{Z}^+$ contains a smallest element. This principle is known as ____________________ principle.
[induction; well-ordering; Lucas; Fibonacci]

19. The number of arrangements of letters of the word DISCIPLINE is ____________________.
[$\frac{10!}{3!}$; $\frac{8!}{3!}$; $\frac{10!}{3!2!}$; $\frac{8!}{3!2!}$]

20. The solution n of the equation $p(n, n-1) = 5040$ is ________.
[3; 5; 7; 11]

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F.M : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Define a homogeneous linear recurrence relation of degree 2 with constant coefficients, and discuss the characteristic root method to find its solution. Also, solve the recurrence relation

$$F_{n+2} = F_{n+1} + F_n,\ n \geq 0$$

with $F_0 = 0,\ F_1 = 1$ using the characteristic root method. [1 + 3 + 3]

OR

Define an ordinary generating function with example. Use generating function method to solve the recurrence relation

$$a_n - 5a_{n-1} + 6a_{n-2} = 0,\ n \geq 2$$

with $a_0 = 1,\ a_1 = -2$. [2 + 5]

2. Let $n \in \mathbb{Z}^+$ and $q_i \in \mathbb{Z}_{\geq 0}$, $1 \leq i \leq t$. Use mathematical induction to prove that for all $x_1, x_2, \cdots, x_t \in \mathbb{R}$

$$(x_1 + x_2 + \cdots + x_t)^n = \sum_{q_1+q_2+\cdots+q_t=n} p(n; q_1, q_2, \cdots, q_t)\, x_1^{q_1}\ x_2^{q_2} \cdots\ x_t^{q_t}$$

Use the above relation to find the coefficient of ab^3cd^2 in the expansion of $(a + 2b - 3c + d)^7$. [5 + 2]

3. What do you mean by a Theorem and meaning to prove a theorem. Discuss indirect proof method to prove a theorem. Use contrapositive method to prove that if n^2 is odd, then n is odd. [2 + 2 + 3]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Using the laws of logic simplify the Boolean expression $(p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ with appropriate reasons.

5. Let H be a subgroup of a group G and $a \in G$. Let $aHa^{-1} = \{\, aha^{-1} \mid h \in H \,\}$. Show that aHa^{-1} is a subgroup of G.

6. Find the general integral solution of the linear Diophantine equation $8x - 27y = 125$.

7. Find the number of integral solutions of the equation $x_1 + x_2 + x_3 = 6$ where $x_i \geq 1$ for all $i,\ 1 \leq i \leq 3$, and write the solutions.

8. Write the sequence generated by the generating function

$$\frac{3x}{(1 + x)^2}$$

9. In how many ways can 30 distinguishable books can be distributed among 3 people A, B, and C so that C receives at least 2 books, B receives at least twice as many books as C, and A receives at least 3 times as many books as B?

OR

Prove that the number of r-permutation of n distinct objects, $r \leq n$, without repetition is

$$\frac{n!}{(n-r)!}$$

SECTION "E"

[5Q. × 2 = 10 marks]

10. The set $G = \{\pm i, \pm 1\}$ of fourth roots of unity forms a group under the operation of multiplication and $H = \{-1, 1\}$ is a subgroup of G. Find all left cosets of H in G.

11. Find the 3-combinations of $\{1 \cdot a,\ 1 \cdot b,\ 3 \cdot c\}$.

12. Find the coefficient of x^5 in $(x + x^2 + x^3 + \cdots)^3$.

13. For all $a, b \in \mathbb{Z}$. Prove that $a|b \Rightarrow a|bx,\ \forall\, x \in \mathbb{Z}$.

14. Find the number of terms in the expansion

$$\left(\sum_{i=1}^{4} a_i\right)\left(\sum_{i=1}^{5} b_i\right)\left(\sum_{i=1}^{6} c_i\right)$$