KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

04 JUN 2019

Level : B.Sc.
Year : III
Time : 2 hrs. 30 mins.

Course : MATH 322
Semester : II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define the intersection (∩) between two non-empty sets. Also, give the recursive definition for the intersection of sets $B_1, B_2, B_3, \ldots, B_n, B_{n+1}$, where $n \in Z^+$, $n \geq 2$ and then prove the generalized associative law for intersection. [1+2+4]

2. Write the second order linear non-homogeneous recurrence relation with constant coefficients. Also, solve the recurrence relation $c_n = 5c_{n-1} - 6c_{n-2} + 8n^2$ where $n \geq 2$ and $c_0 = 4$, $c_1 = 7$. [2+5]

**OR**

Define generating function with example. Also, use it to solve the recurrence relation $d_n = 2d_{n-1} + 1$ with $d_1 = 1$. [2+1+4]

3. Define a group with example. For groups (G, o) and (H, ∗), show that (G × H, • ) is a group where • is a binary operation on G × H defined by
$(g_1, h_1) \bullet (g_2, h_2) = (g_1 o g_2, h_1 * h_2)$. [2+5]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Use the Principle of Mathematical Induction to prove
$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \ldots + n\cdot(n+1) = \frac{n(n+1)(n+2)}{3} \text{ for } n \geq 1.$$

5. Find the gcd of 19875 and 17680 and express it as linear combination of given numbers.

**OR**

State and prove the Division Algorithm Theorem for integers.

6. For subsets A, B and C, prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$, where symbols have their usual meanings.

7. Find the integer solution(s) of $6x + 10y = 108$ for $x, y \geq 0$.

8. For primitive statements p, q, r, show that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$, where the symbols have their usual meanings.

9. For a cyclic group with cardinality $n > 1$, show that G is isomorphic to $(\mathbf{Z_n}, \oplus)$ where the symbols have their usual meanings.

## SECTION "E"

[5 Q. $\times$ 2 = 10 marks]

10. Determine the number of positive integers n where $1 \leq n \leq 2000$ and n is not divisible by 7 or 8.

11. Find $g^{-1}([-5, 5])$ for the real function g defined by $g(x) = \begin{cases} 3x - 5, & x > 0, \\ -3x + 1, & x \leq 0. \end{cases}$

12. Find the inverse of real function $f(x) = 3x - 5$ and also verify your answer.

13. Find the coefficient of $x^5$ in $(1 - 2x)^{-7}$.

14. If $f : A \rightarrow B$ is a function with $B_1, B_2 \subseteq B$. Then, show that $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.