KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc.
Year : III

Course : MATH 322
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date 07 JUL 2023

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. If H is a subgroup of a finite group G. Then the Lagrange theorem states that ________________________.

2. A sequence $0, 0, 0, 2, 4, 6, 8, 10, \dots$ generates the generating function ________________.

3. The recurrence relation $a_n - 7\, a_{n-1} + 10\, a_{n-2} = 0$, and $a_0 = 0, a_1 = 3$ has the solution $a_n =$ ____________________.

4. The number of 2-combinations of $\{\, 3 \cdot a, 2 \cdot b,\ 1 \cdot c \,\}$ is ________________.

5. An "AND" statement is true if, and only if, both components p and q are ______________.

6. The number of 13 - permutations of the 13-combination "PROCRASTINATE" is ______________.

7. The well-ordering principle states that "Every non-empty subset of $\mathbb{Z}^+$ contains a ______________ element".

8. For all $n \in \mathbb{Z}^+,\ n(n+1)(n+2)$ is divisible by ____________________.

9. The rule of inference $(p \wedge (p \Rightarrow q)) \Rightarrow q$ is known as ____________________.

10. Let A and B be subsets of a universal set Ω. Then the duality of $A \cup (A \cap B) = A$ is given by ____________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK THE ANSWER**, by selecting the most appropriate answer from among the given ones.

11. Let $G = \langle a \rangle$ be an infinite cyclic group generated by the element $a \in G$. Then this cyclic group G has only ____________________ generates.
[one; two; three; infinite]

12. For each $n \in \mathbb{Z}^+$, let $P(n)$ denote the number of partitions of n into positive summands. Then $P(4) =$ ______________________.
[2; 3; 4; 5]

13. Let A and B be sets. Then $A \nsubseteq B$ if, and only if ______________________.
[$(\forall x)\ (x \in A \vee x \notin B)$; $(\forall x)\ (x \in A \wedge x \notin B)$;
$(\exists x)\ (x \in A \vee x \notin B)$; $(\exists x)\ (x \in A \wedge x \notin B)$]

14. Suppose H be a subgroup of a group G, and $a \in G$. Then the coset aH is also a subgroup of G if, and only if ______________________.
[$a \in aH$; $aH \neq G$; $aH \neq Ha$; $a \in H$]

15. Let D_n denote the number of derangements of n items. Then $D_4 =$ ______________________.
[1; 2; 9; 44]

16. The number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 10$ is ______________________.
[$C(14,10)$; $C(14,4)$; $C(13,10)$; $C(13,4)$]

17. Let $a, b \in \mathbb{Z}$, and $b > 0$. Then the Division Algorithm states that there exist unique integers q and r such that $a = q\,b + r$ where ______________________.
[$r > 0$; $r \geq 0$; $0 < r < b$; $0 \leq r < b$]

18. The induction hypothesis of Mathematical induction for $1 + 2^n < 3^n$, $n \geq 2$ is that it is assumed that at $n = k$ ______________________.
[$1 + 2^k < 3^k$; $1 + 2^k > 3^k$; $1 + 2^{k+1} < 3^{k+1}$; $1 + 2^{k+1} < 3^{k+1}$]

19. Let p and q be primitive statements. Then the compound statement ______________________ is a tautology.
[$(p \vee q) \Rightarrow q$; $p \vee (q \Rightarrow p)$; $p \vee (p \Rightarrow q)$; $(p \wedge q) \Rightarrow q$]

20. The total number of terms in the Multinomial expansion of $(x_1 + x_2 + \cdots + x_t)^n$ is ______________________.
[$C(t-1+n, n)$; $C(t-1+n, t)$; $C(t+1+n, n)$; $C(t+1+n, t)$]

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SECTION "C"

[3 Q. × 7 = 21 marks]

1. a. Prove that the number of r-combinations of n objects with unlimited repetitions is $C(n-1+r,\ r)$. [4]

   b. How many ways are there to fill a box with a dozen doughnuts chosen from 8 different varieties of doughnuts? [3]

OR

   a. State and prove the multinomial theorem. [4]

   b. Use the multinomial theorem to expand $(x_1+x_2+x_3)^3$. [3]

2. a. Define ordinary and exponential generating functions. [1]

   b. If $n, r \in \mathbb{Z}^+$ and $0 \le r \le n$, prove that $C(-n,r) = (-1)^n\, C(n-1+r,\ r)$, and then show that the Maclaurin's series expansion of $(1+ax)^{-n}$ is equal to $\sum_{r=0}^{\infty} C(-n,r)\, a^r x^r$ where $a \in \mathbb{R}$. [4]

   c. Find the sequence generated by the generating function $\frac{1}{(1+x)^3}$. [2]

3. a. State and prove the stronger form of the principle of mathematical induction. [4]

   b. Use mathematical induction to prove that for $n \ge 1$, $n(n^2+5)$ is an integer divisible by 6. [3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Solve the recurrence relation $a_n - 6\,a_{n-1} + 8\,a_{n-2} = 3^n$ where $n \ge 2$, $a_0 = 3$ and $a_1 = 7$ using the characteristic root method and undetermined coefficients method.

OR

Solve the recurrence relation $a_n - 2\,a_{n-1} = 4^{n-1}$ for $n \ge 1$ and $a_0 = 1$, $a_1 = 3$ using generating function.

5. Simplify with reasons that the compound statement $(p \wedge q) \Rightarrow (p \wedge q)$ is a tautology.

6. Determine the coefficient of x^5 in $(1+2x+x^2)^{10}$.

7. Let H_1 and H_2 be any two subgroups of a group G. Prove that $H_1 \cap H_2$ is also a subgroup of G.

8. Determine that the linear equation $6x + 51y = 21$ is a Diophantine equation, and then find its general solution.

9. Find the number of 3-combinations and 3-permutations of $\{\ 2 \cdot a,\ 1 \cdot b,\ 3 \cdot c\ \}$, and then list all of them.

SECTION "D"

[5 Q. × 2 = 10 marks]

10. Let G be an abelian group and $a, b \in G$. Prove that $(ab)^2 = a^2b^2$.

11. Write an expression for a_r, where a_r is the coefficient of x^r in the generating function $\frac{1}{x^2-3x+2}$.

12. Let $n \in \mathbb{Z}$. Use the contrapositive method to prove that if $3n + 2$ is odd, then n is odd.

13. Find the number of integral solution to $x_1 + x_2 + x_3 + x_4 = 50$ where $x_1 \geq -4,\ x_2 \geq 7,\ x_3 \geq -14,\ x_4 \geq 10$.

14. Let G be a group, and $a, b, c \in G$. Then prove that if $ab = ac$, then $b = c$.