KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

| Marks Scored: |
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Level : B.E.
Year : III

Course : MATH 322
Semester : II

Exam Roll No. :    Time: 30 mins.    F. M. : 10

Registration No.:    Date : 08 AUG 2024

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. A declarative sentence that is either true or false but not both is called a(an) ______.
2. The contrapositive of the statement $p \to q$ is ______.
3. An element $a \in X$ is called the least element of $X$ if ______ for all $x \in X$.
4. How many different arrangements are there of the letters $B, A, L, L$? ______.
5. The number of $r-$ combination of $\{\infty.a_1, \infty.a_2, \cdots, \infty.a_n\}$ is the same as the number of the $r-$ combination of ______.
6. The generating function for the sequence ______ is $f(x) = \frac{1}{1-x}$.
7. The recurrence relation of the sequence $3, 7, 11, 15, \cdots$ is ______.
8. In the Division Algorithm, given any integers $a$ and $b$, with $a > 0$, there exist unique integers $q$ and $r$ satisfying $b =$ ______, $0 \leq r < a$ when $b$ is divided by $a$.
9. $(\mathbb{R}, +)$ is a group with identity element, $e =$ ______.
10. A left coset of a subgroup $H$ of a group $G$ is defined by $aH =$ ______.

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (**DO NOT TICK** THE ANSWER).

11. According to Modus Tollens Law, $[(p \to q) \land \neg q] \to$ ______.
    [$p$    $\neg p$    $q$    $\neg q$]

12. According to the Absorption Law______________________.

[ $A \cup B = B \cup A$ $\quad$ $A \cup (A \cap B) = A$ $\quad$ $A \cup \bar{A} = \Omega$ $\quad$ $A \cup \phi = A$]

13. The number of 3-combination of 5 objects with unlimited repetitions is ______________.

[$C(7,3)$ $\quad$ $C(7,5)$ $\quad$ $C(5,3)$ $\quad$ $C(4,3)$]

14. For any three positive integers $a, b, c,$ the Diophantine equation $ax + by = c$ has ______________solution $x = x_0,\ y = y_0$ if $\gcd(a, b)$ divides $c$.

[ natural $\quad$ integer $\quad$ rational $\quad$ real]

15. The number of solutions of the equation $x_1 + x_2 + x_3 = 5$ with non-negative integer variables $x_1, x_2, x_3$, is ____________________.

[15 $\quad$ 17 $\quad$ 21 $\quad$ 23]

16. The coefficient of $a^5b^2$ in the expansion of $(a + b)^7$ is____________.

[96 $\quad$ 208 $\quad$ 216 $\quad$ 21]

17. The order of a difference equation $2a_{n+1} - 2\,a_n + +5a_{n-2} = 0$ is ______________.

[1 $\quad$ 2 $\quad$ 3 $\quad$ 4]

18. The particular solution of $a_n - 3a_{n-1} = 3^n, n \geq 1$ is $a_n^{(p)} =$ ____________________.

[$3^n$ $\quad$ $3^{n+1}$ $\quad$ $n3^n$ $\quad$ $(n-1)3^n$

19. The permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 \end{pmatrix}$ is ____________________________.

[even $\quad$ odd $\quad$ transposition $\quad$ neither]

20. The order of the generator, $i$ of a cyclic group $G = \{-1, 1, -i,\ i\}$ under the multiplication operation is _____________.

[1 $\quad$ 2 $\quad$ 3 $\quad$ 4]

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05 AUG 2024

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Time : 2 hrs. 30mins.

Course : MATH 322
Semester : II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define open statement and state when it becomes a statement. Show that the two statements $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent. Use laws of logic to simplify the statement $\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$. [2+3+2=7]

2. Discuss the method of undetermined coefficients to solve recurrence relations. Solve the recurrence relation, $a_{n+2} - 8a_{n+1} + 16a_n = 8(5^n) + 6(4^n)$. [2+5=7]

3. If $\phi: \mathbb{R}^* \rightarrow \mathbb{R}^*$ under multiplication is defined by $\phi(x) = |x|$, then show that $\phi$ is a group homomorphism. Also, $\mathbb{Z}_4$ is a group of integer modulo 4 under addition operation, and $G = \{-1, 1, -i, i\}$ is a group under multiplication operation, show that the two groups are isomorphic. [3+4=7]

**OR**

Define a group and a subgroup. Let $H \leq G$ and $K \leq G$, show that $H \cap K \leq G$. Give a counter-example to show that it is not true for $H \cup K$. [2+3+2=7]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. State the principle of mathematical induction. Prove that $3|(n^3 - n)$ for all $n \in \mathbb{Z}^+$.

5. Define permutation and combination. If five friends like to order fruits at a fruit shop that serves APPLES, ORANGES, or BANANAS. Find the number of orders possible.

**OR**

Use the direct and indirect approach to find the number of different arrangements of the letters of the word 'REFERENCE'.

6. Prove the generalized form of DeMorgan's Law: $\overline{\bigcup_{\iota\in I} A_\iota} = \bigcap_{\iota\in I} \bar{A_\iota}$ and $\overline{\bigcap_{\iota\in I} A_\iota} = \bigcup_{\iota\in I} \bar{A_\iota}$.

7. Discuss the Diophantine Equation and find the integer solutions of $10x + 14y = 144$ where $x, y \geq 0$.

8. Use the Euclidean Algorithm to find the gcd$(a, b)$ with $a = 16784$ and $b = 24648$, then express it as a linear combination of $a$ and $b$.

9. Define the order of a group. State and prove Lagrange's theorem for finite groups.

**P.T.O.**

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the negation of the statement, $\forall x[\neg p(x) \rightarrow q(x)]$.

11. Use the direct proof method to show that if $x$ and $y$ are odd, then $xy$ is odd.

12. Determine $b_r$ if $h(x) = \sum_{r=0}^{\infty} b_r x^r = \frac{1}{x^2-5x+6}$.

13. Find the coefficient of $x^9$ in $(1 + x^3 + x^8)^{10}$.

14. Show that $(\mathbb{Z}, +)$ is a subgroup of $(\mathbb{R}, +)$, where the symbols have their usual meanings.