KATHMANDU UNIVERSITY
End Semester Examination
February, 2025

Marks Scored:

Level : B.Sc.
Year : III

Course : MATH 322
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : ~~21~~ 23 24 FEB 2025

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. A map $\phi: \mathbb{Z}_{12} \to \mathbb{Z}_{12}$ defined by $\phi(x) = 4x$ is a group homomorphism. Then the kernel of ϕ, that is, $\ker \phi =$ ____________________.

2. Let $G = \langle g \rangle$ be an infinite cyclic group generated by $g \in G$. Then the number of generators in G is equal to ____________.

3. For a sequence $\{a_r\}_{r=0}^{\infty}$ of real numbers, the function $f(x) = \sum_{r=0}^{\infty} a_r x^r$ is called an __________________ generating function.

4. The coefficient a_r of the generating function $\frac{1}{1-x} + \frac{1}{1+x}$ is ____________.

5. The number of non-negative integral solutions to $e_1 + e_2 + e_3 = 4$ where $e_i \geq 0$ for each i is ____________.

6. The number of terms in the product of the expression $\left(\sum_{i=0}^{5} a_i\right)\left(\sum_{i=-3}^{2} b_i\right)\left(\sum_{i=2}^{7} c_i\right)\left(\sum_{i=-1}^{4} d_i\right)$ is _______________.

7. Consider the statement $P(n): n^2 < 2^n$. Then the smallest positive integer for which $P(n)$ is true is ____________.

8. Let $d = \gcd(a, b)$, and the linear Diophantine equation $ax + by = c$ is solvable. Then ______________.

9. For $i = 1, 2, 3, \cdots$, let $A_i = [-i, i]$. Then $\bigcup_{i=1}^{n} A_i =$ ______________.

10. To prove a theorem means to show that the implication $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \Rightarrow q$ is a ______________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (***DO NOT TICK THE ANSWER***)

11. Every non-empty subset of $\mathbb{Z}^+$ contains a ______________________ element.
[smallest; largest; lattice; zero]

12. According to the principle of mathematical induction, if $P(k+1): a^{k+1} + 7$ is true, then $P(k) =$ ________________ must be true.
[$7\,a^k$; $a^k + 7$, a^k; $a^{k+2} + 7$]

13. Let A and B be any two non-empty sets. Then the law $A \cup (A \cap B) = A$ is known as ___________________ law.
[domination; identity; absorption; inverse]

14. Let $A_i = \left(1 - \frac{1}{i},\ 1 + \frac{1}{i}\right)$. Then $\bigcap_{i=1}^{\infty} A_i =$ ___________________.
[(1, 1); (0, 2); (0, 1); {1}]

15. The set $S = \{1,\ \omega,\ \omega^2\}$ is a set of cube roots of unity, and this set S forms a group under the operation of multiplication. Then the order of ω^2 is _______________.
[1; 2; 3; 4]

16. An injective homomorphism is called a(n) ____________________.
[epimorphism; monomorphism; endomorphism; automorphism]

17. If $p(n)$ denotes the number of the partitioning of a positive integer n into positive summands. Then $p(n) =$ ______________________.
[2; 3; 4; 5]

18. If $n \in \mathbb{Z}^+$, then the series expansion of $\frac{1}{(1+x)^n} =$ ______________________.
[$\sum_{r=0}^{n} C(n+r-1, r)x^r$; $\sum_{r=0}^{n} (-1)^r\, C(n+r-1, r)x^r$;
$\sum_{r=0}^{n} C(n+r-1, n)x^r$ $\sum_{r=0}^{n} (-1)^r C(n+r-1, n)x^r$]

19. The number of r-permutations of n objects with unlimited repetitions is - _________________.
[r^n; n^r; $C(n+r-1, r)$; $C(n+r-1, n)$]

20. The coefficient of x^3y^7 in $(x+y)^{10}$ is ____________________.
[98; 116; 120; 132]

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SECTION "C"
[3Q. × 7 = 21 marks]

1.

a. State and prove the stronger form of mathematical induction. [4]

b. A sequence of numbers $a_1, a_2, a_3, \cdots$ are defined by

$$a_1 = 1, a_2 = 2, \text{ and } a_n = a_{n-1} + a_{n-2}, \ \ n \geq 3$$

Use mathematical induction to prove that $a_n < \left(\frac{7}{4}\right)^n$ for all integers $n \geq 1$. [3]

OR

a. Let $a, b \in \mathbb{Z}^+$. Use mathematical induction to prove the division algorithm that states that if a is divided by b, then there exist unique integers q and r such that $a = q\,b + r, \ \ 0 \leq r < b$. [4]

b. Let $a, b \in \mathbb{Z}^+$, and r a remainder when a is divided by b. Then prove that $\gcd(a, b) = \gcd(b, r)$. [3]

2.

a. State and prove the Lagrange theorem in group theory. [5]

b. If n is a positive integer, and a is relatively prime to n, then $a^{\phi(n)} \equiv 1 \ (\,mod\ n\,)$, where $\phi(n)$ the Euler ϕ-function. This result is known as Euler's theorem. Verify Euler's theorem for $n = 15$, and $a = 4$. [2]

3.

a. Consider a linear non-homogeneous recurrence relation with constant coefficients (LNHRRCC) of degree 2:

$$a_n = c_1 a_{n-1} + c_2\, a_{n-2} + f(n)$$

where c_1 and c_2 are constants. Discuss the method of undetermined coefficients to find particular integral a_n^p of this LNHRRCC. [3]

b. Use the method of undetermined coefficients to solve the following LNHRRCC [4]

$$a_n - 5\,a_{n-1} + 6\,a_{n-2} = 4^n, \ \ n \geq 2, \ \ a_0 = 0, \ \ a_1 = 1$$

P.T.O.

SECTION "D"

[6Q. × 4 = 24 marks]

4. Find the number of integral solutions to $x_1 + x_2 + x_3 = 8$ where$x_1 \geq 1, x_2 \geq 2$, and $x_3 \geq 3$, and list all the integral solutions.

OR

How many integral solutions are there of $x_1 + x_2 + x_3 = 20$ if $1 \leq x_1 \leq 6, 1 \leq x_2 \leq 7$, and $1 \leq x_3 \leq 8$?

5. For $x \geq 0$ and $y \geq 0$, solve the linear Diophantine equation $7x + 5y = 12$.

6. In how many ways can 30 indistinguishable books be distributed among 3 people A, B, and C so that C receives at least 2 books, B receives at least twice as many books as C, and A receives at least 3 times as many books as B?

7. Write an expression for a_r the coefficient of x^r in the generating function $\frac{x+21}{(x-5)(2x+3)}$.

8. Suppose p, q, and r are three primitive statements. Use the laws of logic to simplify

$$\neg p \vee \neg q \vee (p \wedge q \wedge \neg r)$$

with appropriate reasons.

9. $U(10)$ denotes the set of positive integers less than 10 and relatively prime to 10. This set $U(10)$ forms the group under the operation of multiplication modulo 10.
   a. Construct the Cayley Table for $U(10)$.
   b. Determine whether the subgroup $H = \{1,\ 9\}$ is normal in $U(10)$ or not.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Suppose $n \in \mathbb{Z}$. Use the contrapositive method to prove the statement: "If $7n + 5$ is even, then n is odd".

11. Use the Euclidean algorithm approach to find the greatest common divisor of 234 and 72.

12. Find the 3-permutations of $\{3 \cdot a,\ 1 \cdot b,\ 1 \cdot c\}$.

13. Find the coefficient of x^{10} in $(1 + x + x^2 + \cdots)^2$.

14. Show that the two statements $\neg(p \Rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

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SECTION "A"
[20 Q. × 0.5 = 10 marks]

For the questions 1-10, encircle the correct option.

1. As per the rule of significant figures, the final result in the expression 32.01 + 5.325 + 12.5, should be:
   a. 49.800 b. 49.835 c. 49.840 d. 49.8

2. What does a stochastic model describe in surveying?
   a. The exact measurements taken.
   b. The probability of randomness involved in a quantity.
   c. The physical laws governing the measurements.
   d. The fixed parameters of a model.

3. When the samples contain less than 30 values, the probability distribution used is:
   a. t - distribution b. Normal distribution
   c. χ^2 - distribution d. F - distribution

4. Magnitude of dispersion of a quantity is an indicator of:
   a. Precision b. Randomness c. Accuracy d. Regular Nature

5. The quantity whose value is dependent upon the values of one or more quantities is:
   a. Conditioned quantity b. Independent quantity
   c. Observed value of the quantity d. Direct quantity

For the questions 6-10, four statements are provided. Choose the correct option that includes only correct statement(s).

6. Statements:
   i. Leading zeros are not significant, but they indicate the position of the decimal point.
   ii. Trailing zeros in a decimal number are significant, while trailing zeros in an integer are always significant.
   iii. Leading zeros can affect the precision of a measurement when expressed in scientific notation, while trailing zeros in an integer may or may not be significant depending on context.
   iv. All leading zeros are insignificant, and all trailing zeros are significant if they come after a decimal point.
   a. Both i and iii b. Only ii c. Both ii and iii d. Only iv

7. Statements:

i. Least square method applies for both systematic and random errors.
ii. Random errors can be removed through specific corrections.
iii. Redundant observations are inconsistent and can't provide unique solution.
iv. Precision of an observation is independent upon the stability of the environment.

a. Both i, iii and iv b. Only iii c. Both ii and iii d. Both iii and iv

8. Statements:

i. In estimation, the sample estimates such as mean and variance matches the mean and variance of the population.
ii. Mean and variance computed from sample set are considered as random variables.
iii. Higher confidence is placed on a sample set with smaller variance than one with the larger variance.
iv. In statistical inferences, the quoted criterion for the required size of a sample is a size larger than 30.

a. Both iii and iv b. Both ii and iv c. i, ii and iii d. ii, iii and iv

9. Statements:

i. The probability density function of a normal random variable is symmetric around the mean.
ii. Approximately 95% of all measurements lie within two standard deviations from the mean.
iii. The standard normal distribution has a mean of 1 and a variance of 1.
iv. The area under the normal distribution curve is always equal to 1.

a. i ii and iii b. i and iv c. i, ii and iv d. i, iii and iv

10. Statements:

i. Relative worthiness of an observation is measured by its weight.
ii. A more weighted observation implies observation with high variance.
iii. Corrections should be directly proportional to weights.
iv. Weights are used to control the sizes of corrections applied to measurements in an adjustment.

a. i, ii and iv b. i, iii and iv c. i and iv d. Only i

For the questions 11 - 20, fill in the blanks with the appropriate words.

11. The value of 95% error of an observation is given by $\pm$ _______________ σ.

12. If a quantity x of weight w is divided by a factor α, then the resulting weight is equal to ___________.

13. The area between $+\sigma$ and $-\sigma$ equals approximately ________ of the total area under the normal distribution curve.

14. The _____ distribution is used for answering the question of whether two sample sets come from the same population.

15. The required number of control points in a 2D affine transformation are _______.

16. For a rectangular field with sides 85.45 ± 0.012 m and 145.05 ± 0.020 m, the error in the area of the field is ______________ m^2.

17. Formulating a ______________ model is a pre-requisite to perform any adjustment.

18. If the weight of the equation $(A + B) = 58^0 46' 25''$ is 4, then the weight of the equation $180^0 - (A + B)$ would be ________.

19. The semi-major axis of the standard error ellipse exist in the direction of __________ uncertainties of the point.

20. On the normal distribution curve, the closer spacing between the inflection points represents __________ precise observations.