KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

FEB 25 2019

Level : B.Sc.
Year : III
Time : 2 hrs. 30 mins.

Course : MATH 322
Semester: II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define an intersection between two sets. Give the recursive definition of intersection of sets $A_1, A_2, A_3, \ldots, A_n, A_{n+1}$, where $A_i \subset U$, a universal set for all $1 \le i \le n + 1$, and then prove the generalized associative law for intersection. [1+2+4]

2. Write the second order linear homogeneous recurrence relation with constant coefficients and the formulas for its solution. Also, solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$ where $n \ge 0$ and $a_0 = 0$, $a_1 = 1$, $a_2 = 2$. [1+3+3]

OR

Define a ring with example. Also, prove that the set Z of integers with binary operation ⊕ and ⊗ defined by $x \oplus y = x + y - 1$ and $x \otimes y = x + y - xy$ is a commutative ring with unity. [2+1+4]

3. Define a group with example. Also If (G, o), (H, ∗) are groups with respective identities e_G, e_H. If $f : G \to H$ is a homomorphism then show that $f(S)$ is a subgroup of H for each subgroup S of G. [2+1+4]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Determine the number of positive integers n where $1 \le n \le 300$ and n is not divisible by 5, 6 or 8.

5. State the multinomial theorem and then evaluate the coefficient of $x\, y\, z^2$ in $(2x - y - z)^4$.

OR

Apply the method of the generating function technique to solve the recurrence relation $b_n = 3b_{n-1} + n$ where $n \ge 1$ and $b_0 = 1$.

6. For all positive integers a and b, prove that there exists a unique greatest common divisor c of a and b.

7. Find the integer solution of $4x + 6y = 104$ where $x, y \ge 0$.

8. For open statements p, q, r, prove that $(p \vee q) \to r \Leftrightarrow (p \to r) \wedge (q \to r)$, where the symbols have their usual meanings.

9. For any non-empty sets A, B, and C, show that $A \times (B - C) = (A \times B) - (A \times C)$, where symbols have their usual meanings.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Use the principle of mathematical induction to prove that $2^n > n^2$ for $n \geq 4$.

11. Find $f^{-1}([-3, 3])$ for the real function f defined by $f(x) = \begin{cases} 3x-5, & x>0 \\ -3x+1, & x \leq 0 \end{cases}$.

12. Find the partition of $G = (\mathbf{Z}_{12}, +)$ relative to its subgroup $H = \{[0], [4], [8]\}$.

13. Prove that for any positive integer n, 5n+3 and 7n+4 are relatively prime.

14. Show that the product of two odd integers is odd.