Mark Scored:

KATHMANDU UNIVERSITY
End Semester Examination
August/September, 2017

Level : B. Sc.
Year : III

Course : MATH 322
Semester : II

Exam Roll No. :

Time: 30 min

F. M. : 20

Registration No.:

Date : SEP 03 2017

SECTION "A"

[10 Q × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbols(s).

1. A commutative ring with unity is ________________ it has no proper divisor of zero.

2. The generating function for the sequence 1, 2, 4, 8, …, is $f(x) =$ ________________

3. The evaluation of the Boolean expression $\sim [(a \le b) \vee (b > c)$ for a = 3, b = 5 and c = 6, is ________________

4. A function f: A→B is onto if ________________

5. The recurrence relation for the sequence 3, 7, 11, 15, 19, …, is ________________

6. $\lfloor -275.33 \rfloor =$ ________________

7. If GCD(a, b) = 1, then a and b are ________________

8. A semi-group that has an identity element is ____________

9. The subgroup H of a group G defined by H = {a∈G: ag = ga for all g in G} is ________ of G.

10. The set $\boldsymbol{Z}_p$ is ________________________ if p is prime, symbol has usual meaning.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The number of solutions of the equation $x_1 + x_2 + x_3 = 5$ with non-negative integer variables x_1, x_2, x_3, is ________
[15 17 21 23]

12. For two real valued functions f and g defined by $f(x) = x^2 + 1$ and $g(x) = x - 3$, then (gof)(-1) is __________
[- 2 - 1 0 1]

13. For any three positive integers a, b, c, the Diophantine equation $ax + by = c$ has ______________ solution $x = x_0$, $y = y_0$ if gcd(a, b) divides c.
[natural integer rational real]

14. The coefficient of x^2y^2 in the expansion of $(2x - 3y)^4$ is ____________
[96 208 216 224]

15. The relation $a_{n+1} = 3\ a_n + 2\ a_{n-1} - 5\ a_{n-2}$ is a linear homogeneous recurrence relation with constant coefficient of order ________
[1 2 3 4]

16. If g is the mod-7 function, then g(153) = ____________
[3 4 5 6]

17. The symmetric group S_4 of an equivalent triangle under the operation of composition of two permutations is of order ___________
[4 8 16 24]

18. The least common multiple of 252 and 595 is ___________
[21418 21420 21423 21425]

19. The number of prime less than or equal to 100 is ___________
[21 23 25 27]

20. The value of n for which P(n, 2) = 90 is ___________
[5 10 15 20]

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SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define the union (∪) between two non-empty sets. Also, give the recursive definition for the union of sets $A_1, A_2, A_3, \ldots, A_n, A_{n+1}$, where $n \in \boldsymbol{Z}^+$, $n \geq 2$ and then prove the generalized associative law for union. [1+2+4]

2. Write the second order linear non-homogeneous recurrence relation with constant coefficients. Also, solve the recurrence relation $c_n = 5c_{n-1} - 6c_{n-2} + 8n^2$ where $n \geq 2$ and $c_0 = 4$, $c_1 = 7$. [2+5]

OR

Define generating function with example. Also, use it to solve the recurrence relation $b_n = 2b_{n-1} + 1$ with $b_1 = 1$. [2+1+4]

3. Define a group. Also, for given groups (G, o) and (H, $*$) with respective identities e_G and e_H and a homomorphism f from G to H, prove that (i) $f(e_G) = e_H$ and (ii) f(S) is a subgroup of H for each subgroup S of G. [2+2+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. State the principle of strong mathematical induction and use it to prove $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$ for $n \geq 1$.

5. State the multinomial theorem and then evaluate the coefficient of $w^3x^2\,y\,z^2$ in the expansion of $(2w - x + 3y - 2z)^8$.

OR

For positive integers a and b, show that there exists a unique greatest common divisor.

6. For subsets A, B and C, prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$, where symbols have their usual meanings.

7. Find the integer solution(s) of $6x + 10y = 108$ for $x, y \geq 0$.

8. For primitive statements p, q, r, show that $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$, where the symbols have their usual meanings.

9. Use Euclidean Algorithm to find the gcd(a, b) with a = 2076 and b = 1024, and then express it as a linear combination of a and b.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Determine the number of positive integers n where $1 \leq n \leq 2000$ and n is not divisible by 7 or 8.

11. Find $f^{-1}([-5, 5])$ for the function $f : \Re \to \Re$ defined by $f(x) = \begin{cases} 3x - 5, & x > 0, \\ -3x + 1, & x \leq 0. \end{cases}$

12. Find the partition of $G = (\mathbf{Z}_{12}, +)$ relative to its subgroup $H = \{[0], [4], [8]\}$.

13. Find the coefficient of x^5 in $(1-2x)^{-7}$.

14. If $g : A \to B$ is a function with $A_1, A_2 \subseteq A$. Then, show that $g(A_1 \cup A_2) = g(A_1) \cup g(A_2)$.