Marks scored:

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

Level : B.Sc.
Year : III

Course : MATH 302
Semester: I

Exam. Roll No. : Time: 30 mins. F. M. : 20

Registration No.: Date : APR 6 2017

SECTION "A"

[10Q × 1=10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbols(s).

1. The maximum number of linearly independent vectors in the vector space is called.................................

2. If $B = [b_{ij}]_{2\times2}$ with $b_{i,j} = t^{i+j}$, then $\frac{d^2B}{dt^2}$ at t = 1, is ....

3. The rank of a quadratic form remains invariant under .... transformation.

4. A real symmetric matrix is ....

5. Every normal matrix is unitarily .... to a diagonal matrix.

6. The largest of the absolute value of the eigen values of a square matrix is known as ....

7. A real quadratic form $\mathbf{x}^T A\mathbf{x}$ is positive definite if the determinant of A is ....

8. If $A = \begin{bmatrix} i & 1 \\ 1 & -i \end{bmatrix}$, then A^3 is ....

9. A matrix A of order 5×7 has a left inverse if rank(A) is ....

10. A linear transformation L: $V_n \rightarrow V_m$ is .... if its nullity is zero.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The product of characteristic roots of the matrix $A = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix}$ is ....

    [- 3 ; - 4; 3; 4]

12. A total number of inversion in 531642 is ....

[2; 3; 4; 9]

13. The kernel of L**x** = (x -1, log x) is ....

[-1; 0; 1; ∞]

14. The polynomial f(x) = x^2 + 1 is irreducible over the set of ....

[natural; integer; rational; real]

15. The composite transformation L• K for two transformations K and L defined by L(x, y, z) = (y $x + y - z$) and K(x, y) = ($x - y$ y x) at (-1, 1) is ....

[(0, 0); (-1, 0); (1, 0) ; (1, 1)]

16. A matrix A with A^2 = I, an identity matrix, is called ....

[idempotent; nilpotent; involuntary; unit]

17. The quadratic form $q(x) = (x_1 - 2x_2)^2 - x_3^2$ is .... in $\Re^3$.

[positive definite; non-singular; negative definite; indefinite]

18. The determinant of the matrix $\begin{bmatrix} 1+i & 1 \\ i & i \end{bmatrix}$ with $i^2 = -1$, is ....

[- 1 ; 0 ; 1 ; 2]

19. The signature of the vector **x** = (2, 3, 0, 0, 4, 5) in $\mathbf{V}_6$ is ....

[2; 3; 4; 5]

20. The linear transformation L: V → W has a right inverse where V and W being vector space if it is ....

[injective; surjective; into; bijective]

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SECTION "C"

[3Q. × 7 = 21 marks]

1. Define Hermitian matrix with example. How is it related to symmetric matrix? For any complex square matrix A, show that A + A* is Hermitian where A* is the tranjugate of A. [2+2+3]

2. Define invariant polynomial of an m×n λ-matrices $A(\lambda)$ of rank r. Show that two matrices A and B are similar if, and only if they have the same invariant polynomial. [2+5]

OR

State the Binet-Cauchy formula to find the determinant of the product of two matrices. Also, use it to find the determinant of the following product of matrices and verify your result: $\begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 3 & 1 & 2 \\ 3 & 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 1 \\ 4 & 2 & 4 \\ 3 & 4 & 3 \\ 1 & 2 & 1 \end{bmatrix}$. [2+4+1]

3. Define a λ-matrix of order n and degree m over the field. State Cayley-Hamilton Theorem for the characteristic polynomial and use it to find the inverse of $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$. [2+2+3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Compute A^{20} of the matrix $A = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix}$.

5. Find the left quotient and left remainder of $A(\lambda)$ which is divided by $(\lambda I - B)$ where I is identity matrix, $A(\lambda) = \begin{bmatrix} \lambda^3 - \lambda^2 + 2\lambda & 2\lambda^3 + 3\lambda - 1 \\ \lambda^2 - 2\lambda + 1 & 3\lambda^2 + 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$.

6. Find the coordinates of the vector $\mathbf{x} = [5, -1, 2]^T$ in with $\Re^3$ respective to the basis $\mathbf{x}_1 = [1 \;\; 4 \;\; 2]^T$, $\mathbf{x}_2 = [4 \;\; 2 \;\; 1]^T$, $\mathbf{x}_3 = [2 \;\; 1 \;\; 3]^T$.

7. Use the properties of determinant to prove the identity:

$$\begin{vmatrix} bc & ca & ab \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = (a-b)(b-c)(c-a).$$

OR

Prove that every annihilating polynomial of a square matrix is divisible by its minimum polynomial.

8. Show that a λ-matrix $A(\lambda)$ is divisible on the right by $(\lambda I - B)$ if $A_r(B) = 0$ for compatible square matrix B, where symbols have their usual meanings.

9. Use matrix method to find the solution of the system of linear equations: $x_1 + x_2 - x_3 = 2$, $x_1 + 2x_2 + 3x_3 = 1$ and $x_1 + x_2 + x_3 = 0$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Show that the eigen values of a Skew-Hermitian matrix S are purely imaginary.

11. Find the determinant of the matrix $\begin{bmatrix} 2 & 2 & 3 & 3 \\ 2 & 1 & 4 & 2 \\ 3 & 2 & 3 & 2 \\ 3 & 1 & 2 & 4 \end{bmatrix}$

12. Find the bilinear form of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$.

13. Show that $S = \{(x_1, x_2, x_3) : x_1 + x_2 - x_3 = 0\}$ is a subspace of the vector space $\Re^3$.

14. For the set $\Re$ of real numbers, verify that the linear transformation L on $\Re^2$ defined by $L\mathbf{x} = [x_1 - x_2,\ x_1 + x_2]^T$ for $\mathbf{x} = (x_1, x_2) \in \Re^2$ is an isomorphism.