KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

Level : B.Sc.
Year : III
Time : 2 hrs. 30mins.

18 AUG 2024

Course : MATH 302
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define a complex matrix and its tranjugate with examples. Also, for real symmetric matrix M and real skew-symmetric matrix N, prove that A = M +iN is Hermitian. [2+2+3]

2. State and interpret the *Cayley-Hamilton Theorem* for matrices. Also, verify it for the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 4 & 6\end{bmatrix}$. [2+1+4]

OR

Define a λ-matrix of order and degree m over the field with example. Also, show that $A(\lambda)=\begin{bmatrix}\lambda^2-\lambda+1 & 1-\lambda^3\\ 1+\lambda^3 & \lambda^4+\lambda^3+2\lambda+2\end{bmatrix}$ is equivalent to the matrix $B(\lambda)=\begin{bmatrix}\lambda+1 & 0\\ 0 & \lambda^2-\lambda+1\end{bmatrix}$. [2+5]

3. Define the characteristic equation and an eigen value of a square matrix. Also, use the *Diagonalization* process of a matrix to find A^{100} for $A=\begin{bmatrix}1 & 3\\ 2 & 2\end{bmatrix}$. [1+1+5]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Show that the linear transformation T defined on $\Re^2$ by $Lx = L[x_1, x_2]^T = [x_1 - x_2, x_1 + x_2]^T$ is an isomorphism on $\Re^2$.

OR

Show that every quadratic form x^TAx can be reduced to a diagonal quadratic form y^TDy by means of an orthogonal transformation P, where the diagonal elements of D are the eigen values of A.

P.T.O.

5. Use the matrix method to find the solution and verify the answer:

$2x - y + z = -2$, $x - y - 2z = -9$ and $x - 2y - z = 9$.

6. Find the inverse of the matrix A, by the method of partitioning and also verify the answer

where $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

7. For a λ-matrix $A(\lambda) = \begin{bmatrix} \lambda^3 - \lambda & 3\lambda^2 - \lambda \\ -\lambda^2 + 1 & 2\lambda^3 - 1 \end{bmatrix}$, compute its left value $A_l(B)$ at $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

8. Find the determinant of the matrix: $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & -3 & 4 & 1 \\ 3 & 4 & -2 & 1 \\ -4 & 3 & 1 & 2 \end{bmatrix}$

9. Determine whether the following quadratic form $5x^2 + y^2 + 5z^2 + 4xy - 8xz - 4yz$ is positive definite or not.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Verify that the set of vectors $x_1 = (1, 1, -1)$, $x_2 = (1, -1, 1)$ and $x_3 = (-1, 1, 1)$ forms a basis in $\Re^3$.

11. Show that the bilinear form of the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix}$ is equivalent to its canonical form.

12. For any two non-singular matrices A and B of order n, show that $(AB)^{-1} = B^{-1}A^{-1}$, symbols have usual meanings.

13. For any two similar matrices A and B, show that trace(A) = trace(B).

14. Express the matrix $B = \begin{bmatrix} 1 & 1-i & i \\ -i & 1+i & 1+2i \\ 1-3i & 2 & 3 \end{bmatrix}$, i being an imaginary unit, as the sum of Hermitian and skew-Hermitian matrices.

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Marks Scored:

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Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 18 AUG 2024

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. Two permutations have the same ________________ if they are both even or odd.
2. If $A = [\, a_{ij}]$ with $a_{ij} = t^{2i-j}$, then $\frac{d^2A}{dt^2}$ at t = -1, is ________________
3. The square matrix P over a real field is improper if det(P) is ____________
4. A real Hermitian for q(**x**) is positive definite (PD) if ________ when $\mathbf{x} \neq \mathbf{\theta}$, a zero vector.
5. The product of matrix with its transpose is ______________
6. A partitioned matrix with all ___________ elements equal to null matrix is a quasi-diagonal matrix.
7. Two _______________ forms over a field F are equivalent if and only if the matrices of their forms are congruent over F.
8. The signature of the vector **u** = (1, -8, -5, 5, 7, 3, -4) in V_7 is ____________________
9. A matrix $A = [a_{ij}]$ is _____________ if $a_{ij} = 5$, for all i = j.
10. The degree of the λ-matrix associated with a matrix of order 7 is ________________

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), ***DO NOT TICK,*** *by selecting the most appropriate answers from among the given ones.*

11. The trace of the matrix A^2 for $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix}$ is ....

    [20 ; 25; 30; 35]

12. A total number of inversion in a permutation 76853241 is ....

    [20; 22; 24; 26]

13. The tri-diagonal matrix is a matrix with band ....
[-1; 0; 1; 2]

14. The polynomial $f(x) = x^2 + 1$ is .... over the field $\Re$.
[divisible; reducible; irreducible; complex]

15. A square matrix A is said to be .... if $A^2 = I$, an identity matrix.
[symmetric; involutory; idempotent; asymmetric]

16. The sum of the squares of the latent roots of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ is ....
[1; 3; 5; 7]

17. If A and B are matrices of a linear transformation L defined on a vector space **V** with respect to two different bases of a vector space **V**, then there exists a .... matrix Q such that $B = Q^{-1}AQ$.
[singular; non-singular; scalar; identity]

18. The determinant of the matrix $A = \begin{bmatrix} i-1 & 2i \\ i & i+1 \end{bmatrix}$ with $i^2 = -1$, is ....
[- 2; -1 ; 0 ; 1]

19. The cofactor of a_{22} in a matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 1 & 2 \end{bmatrix}$ is ....
[-1; 0; 1; 2]

20. The kernel of $Lx = (\text{Cot}\, x, \text{Sin}\, 2x)$ is ....
[- π/2; 0; π/2; π]