KATHMANDU UNIVERSITY
End Semester Examination
January/February 2024

Level : B.Sc.	Course : MATH 302
Year : III	Semester : I
Time : 2 hrs. 30mins.	F. M. : 55

31 JAN 2024

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define a complex matrix and its tranjugate with examples. For any complex square matrix A, show that A + A* is Hermitian, where the symbol has its usual meaning. [2+2+3]

2. State the Cayley-Hamilton Theorem for the characteristic polynomial. Also, verify it for the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ and use it to find its inverse. [2+2+3]

OR

Define a λ-matrix of order and degree m over the field with example. Also, show that $A(\lambda) = \begin{bmatrix} \lambda^2 - \lambda + 1 & 1 - \lambda^3 \\ 1 + \lambda^3 & (\lambda + 1)(\lambda^3 + 2) \end{bmatrix}$ is equivalent to the matrix $B(\lambda) = \begin{bmatrix} \lambda + 1 & 0 \\ 0 & \lambda^2 - \lambda + 1 \end{bmatrix}$. [2+5]

3. Define a bilinear form from vector space $\mathbf{V_m}$ to $\mathbf{V_n}$ over the same field F and equivalent bilinear forms. Also, show that the bilinear form of the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix}$ is equivalent to its canonical form. [2+2+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Solve the following system of linear equations using the matrix method:
$2x + y + z = 3$, $-x + 2y + 2z = 1$ and $x - y - 3z = -6$.

5. Compute A^{20} of the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$ using the *Diagonalization process.*

OR

Show that every quadratic form $\mathbf{x}^T A \mathbf{x}$ can be reduced to a diagonal quadratic form $\mathbf{y}^T D \mathbf{y}$ by means of an orthogonal transformation P, where the diagonal elements of D are the eigen values of A.

6. Find the matrix of the linear transformation $T:\Re^3 \to \Re^2$ defined by $T(x, y, z) = (2x - y + 3z, x - 2y + 4z)$ relative to bases $\boldsymbol{B_1} = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $\boldsymbol{B_2} = \{(1, 2), (2, 3)\}$.

7. Find the left quotient and left remainder of $A(\lambda)$ which is divided by $(\lambda I - B)$ where I is an identity matrix, $A(\lambda) = \begin{bmatrix} \lambda^3 - \lambda^2 + 2\lambda & 2\lambda^3 + 3\lambda - 1 \\ \lambda^2 - 2\lambda + 1 & 3\lambda^2 + 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$.

8. Prove the identity, using the properties of determinant,

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z).$$

9. Determine whether the following quadratic form $5x^2 + y^2 + 5z^2 + 4xy - 8xz - 4yz$ is positive definite or not.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Verify that the set of vectors $\mathbf{x}_1 = [1 \ \ 1 \ \ 1]^T$, $\mathbf{x}_2 = [1, \ -1, \ 1]^T$ and $\mathbf{x}_3 = [0 \ \ 1 \ \ 1]^T$ forms a basis in $\Re^3$.

11. Show that the eigen value of a Hermitian matrix are purely real.

12. Prove that, for any two non-singular matrices A and B of order n, $(AB)^{-1} = B^{-1}A^{-1}$.

13. If U is the unitary matrix then prove that $|\det U| = 1$.

14. Express the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

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Marks Scored:

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SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. A matrix $A = [a_{ij}]$ is lower triangular if a_{ij} = ...
2. If $A = \lfloor a_{ij} \rfloor_{2\times 2}$ with $a_{ij} = t^{i+j}$, then $\dfrac{d^2A}{dt^2}$ at t = -1, is ....
3. The cofactor of a_{22} in matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -1 \\ 1 & 1 & 2 \end{bmatrix}$ is ....
4. A real Hermitian for q(**x**) is positive definite (PD) if ....when $\mathbf{x} \neq \mathbf{\theta}$, a zero vector.
5. The associated eigen vectors for distinct eigen values of a matrix are ....
6. The adjoint of the symmetric determinant is ....
7. The determinant of a matrix associated with a quadratic form is called ....
8. The signature of the vector **z** = (1, -3, 0, 2, 5, 0, -2) in V_7 is ....
9. A square matrix A order 7 × 5 has a right inverse if rank(A) is ....
10. The degree of the sum of two λ-matrices which are of degree 7 and 5 respectively, is ...

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The trace of the matrix A^2 for $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix}$ is ....

    [20 ; 25; 30; 35]

12. A total number of inversion in 7354232 is ....
[9; 10; 11; 12]

13. The kernel of $Lx = (\log x, x - 1)$ is ....
[-1; 0; 1; ∞]

14. The determinant of a square matrix U over a field of complex number is $e^{i\alpha}$, where α is ....
[natural; integer; rational; real]

15. A square matrix A is said to be .... if $A_2 = I$, an identity matrix.
[symmetric; involutory ; asymmetric; idempotent]

16. The product of the eigen values of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ is ....
[2; 5; 10; 12]

17. If a linear transformation $L:V \rightarrow V$ has matrices A and B with respect to two different bases of a vector space V, then there exists a .... matrix P such that $B = P^{-1}AP$.
[singular; non-singular; scalar; identity]

18. The determinant of the matrix $\begin{bmatrix} i & 1 \\ i & 1+i \end{bmatrix}$ with $i^2 = -1$, is ....
[- 1; 0 ; 1 ; 2]

19. If $A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -7 \\ 5 & 2 \end{bmatrix}$, then BA is ....
[$\begin{bmatrix} 33 & -22 \\ 4 & 13 \end{bmatrix}$; $\begin{bmatrix} -33 & 22 \\ 4 & 13 \end{bmatrix}$; $\begin{bmatrix} 33 & 22 \\ 4 & -13 \end{bmatrix}$; $\begin{bmatrix} 33 & 22 \\ 4 & 13 \end{bmatrix}$]

20. For a matrix A of order 8, the adj($\lambda I - A$), I being an identity matrix, is a regular λ-matrix of degree ....
[6; 7; 8; 9]