Mark Scored:

KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

Level : B.Sc.
Year : III

Course : MATH 302
Semester: I

Exam Roll No. : Time : 30 mins. F. M. : 20

Registration No.: Date : FEB 2 4 2019

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbols(s).

1. The diagonal elements of skew-Hermitian matrix are ....
2. If $A = [a_{ij}]$ with $a_{ij} = t^{i+j}$, then $\frac{d^2A}{dt^2}$ at t = -1, is ....
3. The cofactor of -2 in matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 1 & 2 \end{bmatrix}$ is ....
4. A real Hermitian for q(**x**) is negative definite (PD) if ....when $\mathbf{x} \neq \boldsymbol{\theta}$, a zero vector.
5. The associated eigen vectors for distinct eigen values of a matrix are ....
6. The square matrix is non-singular if its determinant is ....
7. Two quadratic forms over a field F are equivalent if and only if the matrices of their forms are .... over F.
8. The signature of the vector **u** = (1, 7, 0, 5, 4, 0, 3) in $\mathbf{V}_7$ is ....
9. A square matrix B order 7 × 9 has a left inverse if the rank of B is ....
10. The degree of the product of two λ-matrices which are of degree 7, is ....

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The trace of the matrix $A = [a_{ij}]$ where a_{ij} = i + j, is ....
    [20 ; 25; 30; 35]

12. A total number of inversion in 7354232 is ....
[9; 10; 11; 12]

13. The tri-diagonal matrix is a matrix with band ....
[-1; 0; 1; 2]

14. The determinant of a square matrix U over a field of complex number is $e^{i\alpha}$, where α is ....
[integer; rational; real, complex]

15. A square matrix *A* is said to be .... if A_2 = I, an identity matrix.
[symmetric; involutory; idempotent; asymmetric]

16. The sum of the squares of the eigen values of the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$ is ....
[13; 15; 17; 19]

17. If *A* and *B* are matrices of a linear transformation L defined on a vector space V with respect to two different bases of a vector space V, then there exists a .... matrix P such that $B = P^{-1}AP$.
[singular; non-singular; scalar; identity]

18. The determinant of the matrix $\begin{bmatrix} i & 1 \\ i & 1+i \end{bmatrix}$ with $i^2 = -1$, is ....
[- 1 ; 0 ; 1 ; 2]

19. For a matrix of order 11, the adj(λI – A), I being an identity matrix, is a regular λ-matrix of degree ....
[8; 9; 10; 11]

20. Let $\{x_1, x_2, \ldots, x_n\}$ be a basis in the vector space V_n over the field *F*. then, any set of vectors$\{y_1, y_2, \ldots, y_m\}$ are linearly independent in V_n over *F* if ....
[m > n; m < n; m = n; m ≤ n]

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Time : 2 hrs. 30 mins.

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Semester: I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define a Hermitian matrix with example. How is it related to symmetric matrix ? Also, prove that the diagonal elements of a Hermitian matrix are real. [2+2+3]

2. State and prove Cayley-Hamilton Theorem for the characteristic polynomial. Also, verify it for the matrix $A=\begin{bmatrix}3 & 5\\1 & 4\end{bmatrix}$. [2+3+2]

OR

Define a λ-matrix of order and degree m over the field with example. Also, show that
$A(\lambda)=\begin{bmatrix}\lambda^2-\lambda+1 & 1-\lambda^3\\1+\lambda^3 & (\lambda+1)(\lambda^3+2)\end{bmatrix}$ is equivalent to the matrix
$B(\lambda)=\begin{bmatrix}\lambda+1 & 0\\0 & \lambda^2-\lambda+1\end{bmatrix}$. [2+5]

3. Define an eigen vector of a square matrix. Prove that the eigen vectors associate with distinct eigen values of an n-square matrix are linear independent. Also, find the eigen vector(s) of a matrix $A=\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}$. [1+3+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Solve the following system of linear equations using the matrix method:
$x + 2y - z = -5$, $2x - y + z = 6$ and $x - y - 3z = -2$.

5. Show that the linear transformation T defined on $\Re^2$ by $L\boldsymbol{x} = L[x_1, x_2]^T = [x_1 - x_2, x_1 + x_2]^T$ is an isomorphism on $\Re^2$.

Show that every quadratic form $\boldsymbol{x}^T A\boldsymbol{x}$ can be reduced to a diagonal quadratic form $\boldsymbol{y}^T D\boldsymbol{y}$ by means of an orthogonal transformation P, where the diagonal elements of D are the eigen values of A.

6. Find the inverse of the matrix A, by the method of partitioning and also verify the answer where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$.

7. For a λ-matrix $A(\lambda) = \begin{bmatrix} 3\lambda^2 + 1 & 3\lambda^3 + \lambda^2 \\ \lambda^3 + 2\lambda & 2 \end{bmatrix}$, compute its left value $A_l(B)$ at $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

8. Prove the identity, using the properties of determinant,
$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z).$$

9. Determine whether the following quadratic form $5x^2 + y^2 + 5z^2 + 4xy - 8xz - 4yz$ is positive definite or not.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Verify that the set of vectors $\mathbf{x}_1 = [1 \ \ 1 \ \ 1]^T$, $\mathbf{x}_2 = [1, \ -1, \ 1]^T$ and $\mathbf{x}_3 = [0 \ \ 1 \ \ 1]^T$ forms a basis in $\mathfrak{R}^3$.

11. Show that the bilinear form of the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix}$ is equivalent to its canonical form.

12. Prove that, for any two non-singular matrices A and B of order n, $(AB)^{-1} = B^{-1}A^{-1}$.

13. For a unitary matrix U, prove that $|\det U| = 1$.

14. Express the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.