KATHMANDU UNIVERSITY
End-Semester Examination
February/March, 2018

Marks scored:

Level : B.Sc.
Year : III

Course : MATH 302
Semester: I

Exam Roll No.: Time: 30 mins. F.M. : 20

Registration No.: Date MAR 16 2018

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbols(s).

1. A rectangular matrix with a finite number of rows and a single column is __________.
2. If $A = \lfloor a_{ij} \rfloor_{2\times 2}$ with $a_{ij} = t^{i+j}$, then $\frac{dA}{dt}$ at t = 1, is ______________________.
3. The rank of a quadratic form remains invariant under _____________ transformation.
4. A real Hermitian for q(x) is positive semi-definite (PSD) if ________________when $x \neq \theta$, a zero vector.
5. The __________________ matrix D having the eigen values of A as the diagonal element is called spectral matrix for A.
6. A matrix B over a complex field C is called normal if, and only if B _____________ with its tranjugate B*.
7. Two quadratic forms over a field F are ________________ iff the matrices of their forms are congruent over F.
8. The product of characteristic roots of $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ is ______________________.
9. For a matrix A, AA^T is ______________________.
10. A linear transformation **y** = A**x** in $\Re^n$ is isometry if A is _____________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The trace of the matrix $A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & -3 & 2 \\ 2 & 3 & 2 \end{bmatrix}$ is ______________________.

    [-1; 0; 1; 2]

12. A total number of inversion in 413223 is ________________.
[2; 3; 5; 7]

13. The kernel of $L\mathbf{x} = (x, e^x - 1)$ is ________________.
[-1; 0; 1; ∞]

14. The determinant of a square matrix U over a field of complex number is $e^{i\alpha}$, where α is ________________.
[natural; integer; rational; real]

15. The composite transformation L• K for two transformations K and L defined by $L(x, y, z) = (y \quad x + y - z)$ and $K(x, y) = (x - y \quad y \quad x)$ is ________________.
[(0, 0); (x, 0); (y, 0) ; (z, 0)]

16. For a linear transformation T on $\Re^2$ defined as $T(x, y) = (x+y, x)$, $T^{-1}(3, 2)$ is ________________.
[(-2, -1); v(-2, 1) (2, -1); (2, 1)]

17. If a linear transformation $L: V \to V$ has matrices A and B with respect to two different bases of a vector space V, then there exists a ________________ matrix P such that $B = P^{-1}AP$.
[singular; non-singular; scalar; identity]

18. The determinant of the matrix $\begin{bmatrix} 1+i & 1 \\ i & i \end{bmatrix}$ with $i^2 = -1$, is ________________.
[- 1 ; 0 ; 1 ; 2]

19. If A is matrix of order 15, then adj(λI – A), I being an identity matrix, is a regular λ-matrix of degree ________________.
[8; 10; 12; 14]

20. Let $\{x_1, x_2, \dots, x_n\}$ be a basis in the vector space V_n over the field F. Then, any set of vectors $\{y_1, y_2, \dots, y_m\}$ are linearly independent in V_n over F if ________________.
[m > n; m < n; m = n; m ≥ n]

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Year : III
Time : 2 hrs. 30 mins.

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F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define a complex tranjugate and Hermitian matrix with examples. For a square complex matrix A, prove that A + A* is Hermitian. [2+2+3]

2. Discuss the method of obtaining the inverse of a matrix by the principle of portioning. Also, use it to find the inverse of matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 4 & -1 & 4 \end{bmatrix}$ and verify the result. [3+3+1]

OR

Define a row-equivalent canonical (RC) form of a matrix with example. Also, show that two matrices of the same order are equivalent if and only if they have the same rank. [2+1+4]

3. Define a bilinear form from vector space V_m to V_n over the same field F and equivalent bilinear forms. Prove that the bilinear form of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ is equivalent to its canonical form. [2+2+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. State the Cayley-Hamilton Theorem for the characteristic polynomial and verify it for $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

5. For given square matrices A and B of order n, show that $(A+B)^T = A^T + B^T$, where the symbols have their usual meanings.

6. Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix}$.

OR

Show that the matrix $A(\lambda) = \begin{bmatrix} \lambda^2 - \lambda + 1 & 1 - \lambda^3 \\ 1 + \lambda^3 & (\lambda+1)(\lambda^3+2) \end{bmatrix}$ is equivalent to the matrix

$B(\lambda) = \begin{bmatrix} \lambda + 1 & 0 \\ 0 & \lambda^2 - \lambda + 1 \end{bmatrix}$.

7. Find the matrix of T: $\mathfrak{R}^3 \to \mathfrak{R}^2$ defined by $T(x, y, z) = (2x - y + 3z, x - 2y + 4z)$ relative to the bases $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $\{(1, 2), (2, 3)\}$.

8. Use matrix method to find the solution of the system of linear equations:
$x_1 + 2x_2 + 3x_3 = 14$, $2x_1 + 5x_2 + 2x_3 = 18$ and $3x_1 + x_2 + 5x_3 = 20$.

9. Examine whether the real quadratic form $5x^2 + 7y^2 + 8z^2 - 2xy + 4xz + 2yz$ is positive definite or not.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Prove that, for any two non-singular matrices A and B of order n, $(AB)^{-1} = B^{-1}A^{-1}$.

11. Prove that the transformation L defined on $\mathfrak{R}^2$ by $L\boldsymbol{x} = L[x_1 \;\; x_2]^T = [x_1 - x_2 \,,\; x_1 + x_2]^T$ is an isomorphism on $\mathfrak{R}^2$.

OR

Show that every quadratic form $\boldsymbol{x}^T A \boldsymbol{x}$ can be reduced to a diagonal quadratic form $\boldsymbol{y}^T D \boldsymbol{y}$ by means of an orthogonal transformation P, where the diagonal elements of D are the eigen values of A.

13. Show that the set of vectors $\boldsymbol{x}_1 = [1 \;\; 3 \;\; 3]^T$, $\boldsymbol{x}_2 = [-1, \;\; 2, \;\; 2]^T$ and $\boldsymbol{x}_3 = [1 \;\; 1 \;\; 1]^T$ form a basis in $\mathfrak{R}^3$.

14. Prove that the mapping T defined on $\mathfrak{R}^2$ by $T(x, y) = (x + y, x - y)$ is linear.

15. Prove the identity, using the properties of determinant,

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$