

KATHMANDU UNIVERSITY  
End Semester Examination  
March/April, 2017

Marks scored:

Level : B. Sc.  
Year : III

Course : MATH 301  
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : MAR 29 2017

SECTION "A"  
[10 Q × 1=10 marks]

Fill in the blanks by the most appropriate word(s), figures(s) or symbols.

1. If  $p$  and  $q$  are two primary propositions, then their conjunction is the compound proposition which is evaluated as true only if \_\_\_\_\_ and is evaluated as false if \_\_\_\_\_.
2. In matrix representation of relations the domain is given by the \_\_\_\_\_ and range is given by \_\_\_\_\_ that contain at least one non-zero element.
3. In the poset  $(\mathbb{Z}^+, |)$ , the integers 2 and 4 are called \_\_\_\_\_ since  $2 | 4$ , but 3 and 5 are not since neither  $3 | 5$  nor  $5 | 3$ .
4. If  $a = bq + r$  then  $\gcd(a, b) = \gcd(\_\_\_\_, \_\_\_\_)$
5. Consider the propositions  $p$ ,  $q$  and  $r$  defined as (i)  $p$ : You take a course in Discrete Math (ii)  $q$ : You understand logic (iii)  $r$ : You get an A grade on the final examination. In simple form the meaning of the statement  $(p \wedge \sim q) \Rightarrow \sim r$  is \_\_\_\_\_.
6. According to modus ponens, if a proposition  $p$  is true and the compound proposition  $p \Rightarrow q$  is true then \_\_\_\_\_.
7. A graph is said to be connected if \_\_\_\_\_.
8. The minimum number of edges that are to be removed from a graph to make it disconnected is called \_\_\_\_\_.
9. The maximum number of edges in a simple graph of  $n$  vertices is \_\_\_\_\_.
10. A poset  $(P, \leq)$  is called a lattice if every two element subset of  $P$  has both \_\_\_\_\_ and \_\_\_\_\_.

SECTION "B"  
[10 Q. × 1 = 10 marks]

Fill in the blank space by choosing the most appropriate answers from the given choices.

11. Given two statements-  $p$  : It is cold today and  $q$  : The temperature is  $5^{\circ}\text{C}$ . How is the compound statement 'It is neither a cold day nor the temperature is below  $5^{\circ}\text{C}$ ' is represented as \_\_\_\_\_  
(i)  $\sim(p \wedge q)$       (ii)  $\sim(p \vee q)$       (iii)  $(\sim p) \vee (\sim q)$       (iv)  $(\sim p) \wedge (\sim q)$

12. Which of the following relation is reflexive and transitive but not symmetric on set  $A = \{1, 2, 3\}$ ?
- $\{(1, 1), (2, 2), (3, 3), (1, 3)\}$
  - $\{(1, 1), (3, 3), (1, 3), (3, 1)\}$
  - $\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$
  - $\{(1, 2), (2, 3)\}$
13. For a relation  $R$ , on the Cartesian product of two sets  $A$  and  $B$ , the set  $\{a \in A : (a, b) \in R \text{ for some } b \in B\}$  is called \_\_\_\_\_
- Domain of  $R$
  - Range of  $R$
  - Inverse of  $R$
  - Complement of  $R$ .
14. If  $n > 1$  be a composite integer, then there exists a prime  $p$  such that  $p \mid n$  and \_\_\_\_\_
- $p > n$
  - $p \leq \sqrt{n}$
  - $p \leq n$
  - $p > \sqrt{n}$
15. A relation  $R$  on a set  $A$  is \_\_\_\_\_ if for all  $a, b \in A$ ,  $aRb$  and  $bRa \Rightarrow a = b$ .
- symmetric
  - anti-symmetric
  - asymmetric
  - reflexive.
16. In a graph, the number of vertices is called its \_\_\_\_\_.
- size
  - order
  - degree
  - sub-graph
17. In a graph finite alternating sequence of vertices and edges in which neither edges nor vertices are repeated and start point and end points are different is called \_\_\_\_\_.
- trail
  - circuit
  - path
  - cycle.
18. \_\_\_\_\_ is the algorithm that is used to determine the shortest path in a graph without weight.
- Dijkstra's algorithm
  - Breadth-First-Search algorithm
  - Warshall's algorithm
  - Prim's algorithm
19. Which of the following matrices is symmetric? \_\_\_\_\_
- Adjacency matrix of undirected graph
  - Adjacency matrix of directed graph
  - Incidence matrix of undirected graph
  - Incidence matrix of directed graph
20. Which of the following is not a poset? \_\_\_\_\_
- $(Z, \geq)$
  - $(P(S), \subseteq)$
  - $(Z^+, |)$
  - $(Z, |)$

Level : B. Sc.  
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Time : 2 hrs. 30 mins.

Course : MATH 301  
Semester : I  
F. M. : 55

SECTION "C"

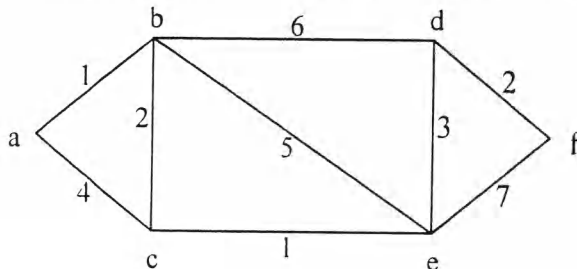
[3 Q. × 7 = 21 marks]

1. Let  $A = \{1, 2, 3, 4\}$  and consider the relation  
 $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 2), (3, 3), (4, 2), (4, 3), (4, 4)\}$   
 Show that  $R$  is a partial ordering set and draw its Hasse diagram describing every steps involved. [2+5]

OR

Draw the diagraph of the relation  $R = \{(2, 1), (2, 2), (2, 3), (3, 2), (3, 4), (4, 4), (4, 5)\}$ . Use Warshall's algorithm to find transitive closure of relation  $R = \{(1, 2), (2, 3), (3, 3)\}$  on  $\{1, 2, 3\}$  describing the steps involved. [2+5]

2. When are the two integers  $a$  and  $b$  said to be congruent? Prove "If  $a$  is an integer which divides  $m$ , then for some integers  $c$  and  $d$ ,  $ac \equiv ad \pmod{m} \Leftrightarrow c \equiv d \pmod{\frac{m}{a}}$ ". Find remainder when the sum  $1! + 2! + 3! + 4! + \dots + 99! + 100!$  is divided by 12. [1+3+3]
3. Draw a graph which has a Hamiltonian circuit but not an Eulerian circuit. Use Dijkstra's algorithm to determine the shortest path in following weighted graph: [2+5]



SECTION "D"

[6 Q. × 4 = 24 marks]

4. (a) Prove using algebra of logics – "If I get the job and work hard, then I will be promoted. If I get promoted, then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard".  
 (b) State converse and inverse of the statement- "If today is not Monday, then tomorrow is not Easter".
5. (a) Let  $U = \{1, 2, 3, 4, 5\}$ ,  $A = \{2, 3, 5\}$  and  $B = \{3, 5\}$ . Find the bit string of  $A$  and  $B$  and then of  $A - B$ .  
 (b) Find the complement of the fuzzy set  $A = \{(x_1, 0), (x_2, 0.3), (x_3, 0.5)\}$
6. Construct truth table of statement  $(\sim (p \wedge q) \vee r) \Rightarrow \sim p$

7. Let  $A = (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, |)$  be a poset. Determine greatest lower bound (glb) and least upper bound (lub), if exists, for following subsets of  $A$  – (i)  $\{1, 2, 3\}$  (ii)  $\{1, 2, 4\}$
8. Prove that in a binary tree the number of internal vertices is one less than the number of pendant vertices.
9. Draw the digraph corresponding to the given adjacency matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

SECTION "E"

[5 Q.  $\times 2 = 10$  marks]

10. Using truth table prove that  $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
11. If  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{4, 5, 6, 7\}$ , find  $(A \Delta B) \Delta B$ .
12. Let  $R = \{(1, 1), (2, 1), (3, 2)\}$ , compute  $R \circ R$ .
13. What is the value of following prefix expression:  $+ - ^3 2 ^ 2 3 / 8 - 4 2$ .
14. Draw a binary tree to represent the algebraic expression  $(x + y * z) - (u / v + w)$