KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2018

Marks Scored:

Level : B.Sc.
Year : III

Course : MATH 301
Semester: I

Exam. Roll No.: Time: 30 mins. F.M. : 20

Registration No.: Date MAR 05 2018

## SECTION "A"

[10 Q.×1=10 marks]

Fill in the blanks by the most appropriate word(s), figure(s) or symbol(s).

1. Consider the logical statements $p$: *Today is Wednesday*, $q$: *It is raining*, and $r$: *It is cold*. The meaning of expression $(p \vee q) \Leftrightarrow r$ is ________________________________________________.

2. If p and q are true and r is false, then the truth value of $(p \vee q) \vee r$ is __________________.

3. Let an and b be two integers with b > 0, then according to division theorem there exists integers q and r with 0 < r ≤ b, such that a = ____________________________.

4. $[4](mod\ 5) + [3]\ (mod\ 5) =$ __________________________

5. Let $A_n = [n, n+1], n \in Z$, then $A_2 \cup A_4 =$ ____________________________.

6. Let A be a fuzzy set defined as $A = \{(x_1, 0), (x_2, 0.3), (x_3, 0.5)\}$, then the complement of A is given by __________________________.

7. A relation R on a set A is called partially ordering if it is reflexive, transitive and __________________________.

8. In the poset $(Z^+, |)$ the integers 3 and 5 are not comparable since _____________________________________________.

9. Let $A = (\{1, 2, 3, \ldots\ldots\ldots\ldots\ldots 10\}, /\ )$ be a poset then in subset {1, 2, 3}, the least upper bound is __________________________.

10. A vertex v in a graph is said to reachable from the vertex u if there exists a _____________________________________________________.

## SECTION "B"

[10 Q.×1=10 marks]

Fill in the blank space by choosing the most appropriate answers from the given choices.

11. Which rule of inference is used in verifying that $(p \Rightarrow q) \wedge p \Rightarrow q$?
    (i) modus ponens (ii) modus tollens
    (iii) hypothetical syllogism (iv) disjunctive syllogism

12. Let "k(x) : x is a student" and " l(x): x is clever." then the statement that "some students are clever" is denoted by ____________________________________
    (i) $\exists\, x\, (k(x) \wedge l(x))$ (ii) $\forall x\, (k(x) \wedge l(x))$
    (iii) $\forall x\, (k(x) \Rightarrow l(x))$ (iv) $\exists\, x\, (k(x) \vee l(x))$.

13. If $\gcd(a, b) = d$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) =$ ________________.

(i) a (ii) b (iii) d (iv) 1

14. Let $A_1, A_2, \ldots\ldots\ldots, A_n$ be subsets of a set A, with (i) $A_1 \cup A_2 \cup \ldots\ldots\ldots \cup A_n = A$ and (ii) $A_i \cap A_j = \emptyset, for\ i \neq j$, then $A_1, A_2, \ldots\ldots\ldots, A_n$ are called ______________ of A.

(i) union (ii) intersection (iii) partition (iv) difference

15. If A = {1, 2, 3, 4, 5} and B = {4, 5, 6, 7} then $(A \cup B)\,\Delta A =$ ______________

(i) {1, 2, 3, 4} (ii) {3, 4, 5} (iii) {6, 7} (iv) {1, 2, 3, 4, 5}

16. Let P = {(1, 2), (2, 4), (3, 3)} and Q = {(1, 3), (2, 4), (4, 2)} then dom(P ∪ Q) = ________________.

(i) {1, 2, 3} (i) {1, 2} (iii) {2, 3, 4} (iv) {1, 2, 3, 4}

17. Which of the following relations represent a poset ? ______________

(i) R = {(a, b) ∈ Z × Z : |a – b| ≤ 1} (ii) R = {(a, b) ∈ Z × Z : |a| ≤ | b|}

(iii) R = {(a, b) ∈ Z × Z : a | b } (iv) R = {(a, b) ∈ Z × Z : a – b ≤ 0}

18. In a graph, the special type of walk in which all edges and vertices are distinct, except the first and last vertices is called ____________________

(i) path (ii) circuit (iii) trail (iv) cycle

19. A geometric representation of graph redrawn in such way that no two of its edges intersect except only at the common vertex is called __________________________

(i) subgraph (ii) regular graph (iii) planar graph (iv) isomorphic graph

20. In a binary tree with n vertices the number of pendant vertices is __________.

(i) n (ii) (n + 1)/2 (iii) n + 1 (iv) n – 2

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SECTION "C"
[3 Q.×7=21 marks]

1. Use prime factorization method to find gcd and lcm of $(150, 70)$. Let a and b be two positive integers, then prove that

$$\gcd(a, b) \times lcm(a, b) = a \times b$$

Solve linear congruence $8x \equiv 2 \ (mod\ 9)$ [1+3+3]

2. Let R be the relation on the set of integers defined by

$$R = \{(x, y): 6 \mid (x - y)\}$$

Prove that the relation is reflexive, symmetric and transitive. Consider the relation $R = \{(0,1), (1,2), (2,3)\}$ defined on $A = \{0,1,2,3\}$. Find reflexive closure, symmetric closure and transitive closure of R by using graphical method. [3+4]

OR

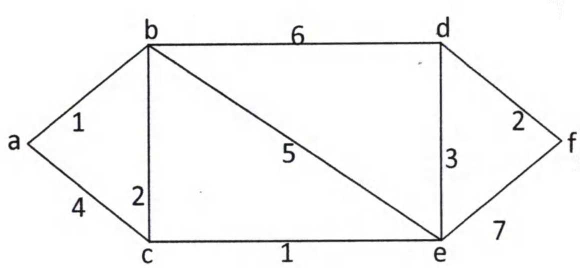
(a) Define composition of two functions $f: A \to B$ and $g: B \to C$. If the function $f: R \to R$ be defined by $f(x) = x^2 + 1$, then find $f^{-1}(-8)$ and $f^{-1}(17)$. [3]

(b) Use Warshall's algorithm to find the transitive closure of relation $\{(1,2), (2,1), (2,3), (3,4), (4,1)\}$ on $\{1,2,3,4\}$. [4]

3. Draw the graph represented by following adjacency matrix and use the matrix to verify that the graph is connected.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Use Dijkstra's algorithm to determine the shortest path in following weighted graph: [3+4]




SECTION "D"
[6 Q.×4=24 marks]

4. (a) Using truth table show that a conditional statement $a \Rightarrow b$ is equivalent to its contraspositive $\sim b \Rightarrow \sim a$.

(b) Prove using 'Logics of Proof' that "If $(p \wedge q) \vee (r \Rightarrow s)$, $t \Rightarrow r, \sim (p \wedge q)$, then $t \Rightarrow s$ is valid."

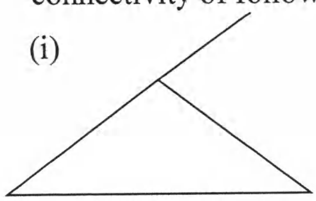
5. Let A and B be two fuzzy sets defined as $A = \{(4, 0.1), (6, 0.5), (8, 0.6), (10, 0.7)\}$ and $B = \{(4, 0.2), (6, 1), (8, 0.4), (10, 0.5)\}$, find $A \cup B$, $A \cap B$, $A^C$ and A – B.

6. Prove that composition of functions is associative, i.e., prove that if $f: A \rightarrow B$, g: $B \rightarrow C$ and h: $C \rightarrow D$, then $(h \circ g) \circ f = h \circ (g \circ f)$.

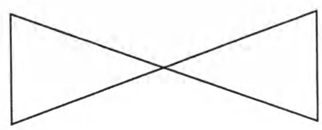
7. Given the relation R on $\{1, 2, 3, 4\}$ defined by $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 2), (3, 3), (4, 2), (4, 3), (4, 4)\}$. Show that R is partially ordering and draw its Hasse diagram.

8. Define (i) edge connectivity and (ii) vertex connectivity of a graph. Idenify the edge connectivity of following graphs-

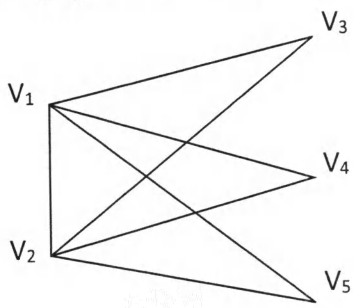
(i)



(ii)



9. What type of graph is called Eulerian graph? What condition on degree of vertices is satisfied by Eulerian graph? Hence identify whether following graph is Eulerian-




SECTION "E"
[5 Q.×2=10 marks]

10. State the rule of 'modus tollens' used in logical inference with an example.

11. Translate following statement into logical expressions – (a) every person is precious (b) some students of this college passed MBA entrance examination.

12. Let A = {2, 3, 4} and B = {3, 4, 5}. List the elements of relation aRb, if a and b are both odd numbers. Also state the domain and range of R.

13. Let $A = \{1, 2, 3, 4, 6, 8, 12, 18, 24\}$. Let R be a relation defined by $x \mid y$, where $x, y \in A$. List elements of R and show that it is partially ordering.

14. Draw the binary tree to represent the given expression and convert the expression in prefix form : $(x + 3y)^5 (a - 2b)$