

KATHMANDU UNIVERSITY
End Semester Examination [C]
January, 2019

Marks scored:

Level : B.Sc.
Year : II

Course : MATH 217
Semester: II

Exam Roll No.:

Time: 30 mins.

F.M. : 20

Registration No.:

Date

JAN 03 2019

SECTION "A"
[10 Q.×1=10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The Maclaurin's series expansion of e^x is _____.
2. The Wronskian of two solutions $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ of a system of two linear equations $\frac{dx}{dt} = ax + by$ and $\frac{dy}{dt} = cx + dy$ is $W(t) =$ _____.
3. The order of Airy's equation $u_t + u_{xxx} = 0$ is _____.
4. The model equation for rate of disintegration of a radio-active substance is given by $\frac{dA}{dt} = -kA, k > 0, A(0) = A_0$. Its solution is $A(t) =$ _____.
5. A simple pendulum consists of a mass m hanging from a string of length l and fixed at a pivot point. When displaced at an initial angle and released, the pendulum will swing back and forth under the resistance of damping force. The model equation is $\frac{d^2\theta}{dt^2} + \frac{\lambda}{m} \frac{d\theta}{dt} + \frac{g}{l} \sin \theta = 0$, where λ is a damping constant. If the displacement $\theta \ll 1$, then the model equation reduces to _____.
6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 = \sin x$ is _____.
7. A Lagrange equation is $y = xf(p) + g(p)$, $p \equiv \frac{dy}{dx}$. If α be the root of the equation $f(p) - p = 0$, then the singular solution of the Lagrange equation is _____.
8. Integrating factor of $\frac{dr}{d\theta} = 500 \theta^n - \frac{r}{\theta}$ is _____.
9. A Bernoulli equation $\frac{dy}{dx} + p(x)y = q(x)y^n$ is linear and non-homogeneous if $n =$ _____.
10. The general solution of the equation $\frac{d^2y}{dx^2} + 4y = 0$ is $y(x) =$ _____, where A and B are arbitrary constants.

SECTION "B"

[10 Q.×1=10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (Do not tick the answer).

11. The second order linear differential equation of the form $\frac{d}{dx} \left\{ (1-x^2) \frac{dy}{dx} \right\} + n(n+1)y = 0$ is _____.
[Rodrigue; Bessel; Lagrange; Legendre]
12. The solution $(x(t), y(t))$ of the system of two equations $\frac{dx}{dt} = 2x$, $\frac{dy}{dt} = y$ is _____.
[(e^{-2t}, e^{-t}) ; (e^{2t}, e^t) ; (e^t, e^{2t}) ; (e^{-t}, e^{-2t})]
13. The Burger's equation $u_t + c u u_x = 0$, where c is a constant is _____ equation.
[non-linear; linear; second order; first order and linear]
14. A stone is dropped from rest at an initial height h above the surface of the earth. Then the speed with which it strikes the ground is _____, where g is the acceleration due to gravity.
[gh ; \sqrt{gh} ; $2gh$; $\sqrt{2gh}$]
15. In an RLC-circuit, we say that the circuit is overdamped if _____.
[$R^2 - \frac{4L}{C} > 0$; $R^2 - \frac{4L}{C} = 0$; $R^2 - \frac{4L}{C} < 0$; $R^2 - 4LC = 0$]
16. A differential equation $\frac{dy}{dx} = f(x, y)$ is said to be in a variable separable form if it can be reduces to the form _____.
[$f(x)dy = g(y)dx$; $\frac{dy}{dx} = \frac{f(x)}{g(y)}$; $f(x, y)dx = dy$; $f(x, y)dy = dx$]
17. A differential equation is considered to be ordinary if it has _____.
[one dependent variable; one independent variable; more than one dependent variable; more than one independent variable]
18. If $\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1000}{x}$, then integrating factor is _____.
[x^{-1000} ; x^{1000} ; x^{-100} ; x^{100}]
19. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{x} (2 - y)$ is $y(x) =$ _____.
[$\frac{C}{x} + 2$; $\frac{C}{x} - 2$; $\frac{2}{x} + C$; $\frac{2}{x} - C$]
20. According to the modification rule in the method of undetermined coefficients, the particular solution $y_p(x)$ of the non-homogeneous differential equation $y''' - 3y'' + 3y' - y = e^x$ is _____, where A is a constant to be determined.
[Axe^x ; Ax^2e^x ; Ax^3e^x ; Ax^4e^x]

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JAN 03 2019
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Semester: II
F.M. : 55

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Time : 2 hrs. 30 mins.

SECTION "C"

[3 Q.×7=21 marks]

1. Solve the 1D wave equation $u_{tt} = 9u_{xx}$, $0 < x < 2$, $t > 0$ subjected to the boundary conditions $u(0, t) = u(2, t) = 0$, $t \geq 0$ and initial conditions $u(x, 0) = x(x -$

$$2), u_t(x, 0) = \begin{cases} 0, & 0 \leq x < \frac{1}{2} \\ 3, & \frac{1}{2} \leq x \leq 1 \\ 0, & 1 < x \leq 2 \end{cases}$$

[4 + 3]

OR

Derive the 1D heat equation $u_t = c^2 u_{xx}$, where c^2 is the thermal diffusivity of a conducting material.

[7]

2. Suppose $P_n(x)$ be a Legendre polynomial of degree n . Establish the Rodrigue formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n, \quad n = 0, 1, 2, \dots$$

Use this formula to find the relation for $P_2(x)$.

[5 + 2]

3. State and prove the linearity principle of a second order homogeneous differential equation. If y_1 and y_2 are two solutions of a second order homogeneous differential equation $y'' + a(x)y' + b(x)y = 0$ on an interval I , and $a(x)$ and $b(x)$ are continuous on I , then show that the Wronskian of the two solution is $W(y_1, y_2)(x) = c e^{-\int a(x) dx}$ where c is a constant depending on only y_1 and y_2 but not on x .

[3 + 4]

SECTION "D"

[6 Q.×4=24 marks]

4. Find the steady-state solution of a mass spring balance system when mass, damping constant and spring constant values are 4, 4, and 17, respectively, and the driving force is $202 \cos 3t$.
5. Find the general solution of $x p^2 + (y - x)p - y = 0$ where $p \equiv \frac{dy}{dx}$.

OR

Find the general and singular solution of the non-linear differential equation $= 2 x p - 3 p^2$, $p \equiv \frac{dy}{dx}$.

6. Solve the non-homogeneous system of linear equations $\frac{dx}{dt} = 3x + 3y + t$, $\frac{dy}{dt} = -x - y + 1$ by elimination method.
7. Find the solution of the initial value problem $y'' - 8y' + 16y = 0$, $y(0) = 1$, $y'(0) = 6$.
8. Use the Laplace transform method to solve the initial value problem $y'' - 4y = 0$, $y(0) = 1$, $y'(0) = 2$.
9. When the air temperature is $70^{\circ}F$, an object cools from $170^{\circ}F$ to $140^{\circ}F$ in 0.5 hour. What is the temperature after 1 hour?

SECTION "E"
[5 Q.×2=10 marks]

10. Use power series method to solve the differential equation $y' = y$.
11. Classify the PDE $u_{xx} + 2u_{xy} + 4u_{yy} + 7u_x + u(x, y) = 0$.
12. Find the solution $u(x, y)$ of the PDE $u_x = u$.
13. Find the integrating factor of $x \log x \frac{dy}{dx} + y = 2 \log x$.
14. Is the differential equation $\cos(x + y) dx + (3y^2 + 2y + \cos(x + y)) dy = 0$ exact?