KATHMANDU UNIVERSITY
End Semester Examination
May/June, 2022

Mark Scored:

Level: B.Sc.
Year : II

Course : MATH 217
Semester : II

Exam Roll No:

Time: 30 mins.

F.M : 20

Registration No:

Date :

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The solution of the initial value problem $xy' = -y,\ y(1) = 1$ for all $x \neq 0$ is $y(x) =$ _____.

2. The function $u(x,t) = 4x^2 + t^2$ is a solution of the 1D wave equation $u_{tt} = c^2\, u_{xx}$ when $c =$ _____.

3. The growth rate of a bacteria population is proportional to its size. Initially the population is 10,000 and after 10 days it is 25,000. Then the population size after 20 days is _____.

4. The eigenvector corresponding to an eigenvalue $\lambda = 1$ of the matrix $A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ is _____.

5. The ODE with variable coefficients $x^2y'' + axy' + by = 0$ where a and b are constants is known as _____ differential equation.

6. The second order linear PDE $y\,u_{xx} + (x+y)u_{xy} + x\,u_{yy} = 0$ is parabolic if _____.

7. The second order ODE in dependent variable x of the system of two linear equations $x' = a_{11}x + a_{12}y,\ \ y' = a_{21}x + a_{22}y$ is _____.

8. The Wronskian of x^2 and x^3 is _____.

9. The degree of the differential equation $\frac{d^3y}{dx^3} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^3}$ is _____.

10. The integrating factor of the non-exact differential equation $3(y+1)dx + 2x\,dy = 0$ is _____.

SECTION "B"

[10Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK THE ANSWER**, by selecting the most appropriate answer from among the given ones.

11. The PDE $yu_{xx} + 2xy\,u_{xy} + xu_{yy} = u_x + u_y$ is hyperbolic on the region _____.
[$xy \neq 1$; $xy \neq 0$; $xy > 1$; $xy > 0$]

12. The singular solution of the Clairaut's equation $y = px + p^2$, $p \equiv \frac{dy}{dx}$ is $y =$ _____.
[$\frac{x^2}{2}$; $\frac{x^2}{4}$; $\frac{3x^2}{4}$; $\frac{3x}{4}$]

13. The solution curve of the initial value problem $\frac{dy}{dx} = -\frac{x}{y}$, $y(4) = -3$ is a(n) _____.
[circle; ellipse; hyperbola; straight line]

14. A Bernoulli equation $\frac{dy}{dx} + ay = by^n$ reduces to a logistic equation if $n =$ _____, where a and b are constants.
[0; 1; 2; 3]

15. The PDE $u_x + uu_y = u$ is _____.
[linear; semilinear; quasilinear; fully linear]

16. If y_1 and y_2 are any two solutions of a second order homogeneous differential equation $y'' + a(x)y' + b(x)y = 0$. Then the linear combination of y_1 and y_2 is also its solution. This principle is known as _____ principle.
[Lagrange; Abel; Superposition; Euler]

17. If the characteristic equation of a differential equation $y'' + ay' + by = 0$ has double roots, say λ. Then the general solution is $y(x) =$ _____, where c_1 and c_2 are constants.
[$c_1e^{\lambda x} + c_2e^{\lambda x}$; $c_1e^{\lambda x} + c_2x\,e^{\lambda x}$;
$c_1e^{\lambda x} + c_2 \log x\, e^{\lambda x}$; $c_1 \cos \lambda x + c_2 \sin \lambda x$]

18. The pairs $(-2e^{-4t}, e^{-4t})$ and $(3e^t, e^t)$ are solutions of the system $X' = AX$ where $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 6 \\ 1 & -2 \end{bmatrix}$. These two solutions are _____.
[linearly dependent; linearly independent;
both; neither]

19. An ODE $y'' + ay' + 6y = 0$ has linearly independent solutions e^{2x} and e^{3x}. Then $a =$ _____.
[-6; -5; 5; 6]

20. The PDE $u_{xx} + u_{yy} = 0$ is known as _____ equation.
[Laplace; wave; heat; D'Alembert]