KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 217
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date 17 JUL 2023

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. The order of the differential equation $(y''')^{3/2} + x\,y' = (1 + x^2)\,y'$ is ________________.

2. The Wronskian of x, x^2, and x^3 is ______________________.

3. The general solution of the differential equation $y''' - y' = 0$ is $y(x) =$ ____________________.

4. The characteristic equation of the 1D wave equation $u_{tt} = c^2\,u_{xx}$ is ______________________.

5. The solution obtained from the general solution by giving a particular value to the arbitrary constant or constants is called a ____________________ solution.

6. The general solution of the system of differential equations $\frac{dx}{dt} = 2x,\ \frac{dy}{dt} = 3y$ is ________________.

7. The function $u(x,t) = e^{-\pi^2 t}\sin 4x$ satisfy the 1D heat equation $u_t = c^2 u_{xx}$ when $c =$ __________.

8. If $y_{p_1} = x$ is a particular solution of $y'' + y = x$ and $y_{p_2} = x^2 - 2$ is a particular solution of $y'' + y = x^2$, then the particular solution of $y'' + y = x^2 + x$ is ____________________.

9. A second-order partial differential equation $u_{xx} + u_{yy} = f(x,y)$ is known as a ____________________ equation.

10. A differential equation of the form $\frac{dy}{dx} + p(x)\,y = q(x)\,y^n$, where n is a rational number is called a ______________________ differential equation.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK THE ANSWER**, by selecting the most appropriate answer from among the given ones.

11. The ordinary differential equation $(ax + by)dx + (cx + ey)\,dy = 0$ is exact if it satisfies the condition ______________________.
[$a = e$; $a = c$; $b = c$; $b = e$]

12. The ordinary differential equation $k\frac{d^2y}{dx^2} = f(x)\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$ is a ______________________
[non-linear first order of degree 2; non-linear second order of degree 2; non-linear first order of degree 3; non-linear second order of degree 3]

13. The singular solution of Clairaut's equation $y = px + p^2$, where $p \equiv \frac{dy}{dx}$ is given by $y(x) =$ ______________, where c is a constant.
[$cx + c^2$; $cx + c$; $\frac{x^2}{4}$; $-\frac{x^2}{4}$]

14. A second-order differential equation of the form $x^2y'' + axy' + by = 0,\ x \neq 0$ is known as ______________________.
[Lagrange; Bessel; Rodrigue; Euler-Cauchy]

15. In an experiment of bacteria culture, the rate of growth of bacteria is proportional to the number of bacteria present at a time. If x is the number of bacteria present at any time t, then ______________________ when the proportionality constant is equal to 2.
[$x(t) = -c\,e^t$; $x(t) = c\,e^{2t}$; $x(t) = c\,e^t$; $x(t) = -c\,e^{2t}$]

16. For the method of undetermined coefficients, the assumed form of the particular integral y_p for $y'' - y = 1 + e^x$ is ______________________, where A and B are constants.
[Be^x; Bxe^x; $A + Be^x$; $Ax + Bxe^x$]

17. The second-order differential equation in $x(t)$ for a system of two linear equations $\frac{dx}{dt} = x + \frac{1}{2}y, \frac{dy}{dt} = 2x + y$ is ______________________.
[$x'' + 2x = 0$; $x'' - 2x = 0$; $x'' + 2x' = 0$; $x'' - 2x' = 0$]

18. The solution $u(x, y)$ of the partial differential equation $u_y = u$ is ______________.
[$c\,e^y$; $c(x)e^y$; $c\,e^x$; $c(y)e^x$]

19. The partial differential equation $u_{xx} + u\,u_y = \sin x + \cos y$ is a ______________________.
[linear equation of order two; non-linear equation of order two; linear equation of order one; non-linear equation of order one]

20. The LR-circuit model equation is ______________________ when emf is $E(t)$.
[$L\frac{dI}{dt} + RI = 0$; $LI + R\frac{dI}{dt} = 0$; $L\frac{dI}{dt} + RI = E(t)$; $LI + R\frac{dI}{dt} = 0$]

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Semester : II
F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. a. If y_1 and y_2 are two solutions of $y'' + a(x)y' + b(x)y = 0$ on an interval I, and $a(x)$ and $b(x)$ are continuous on I, then show that the Wronskian of these two solutions are $W(y_1, y_2)(x) = C\exp\left(-\int a(x)\,dx\right)$
where C is a constant. This relation is known as Abel's formula. [3]

b. Also, show that for any $x_0 \in$ I, [2]

$$W(y_1, y_2)(x) = W(y_1, y_2)(x_0)\exp\left(-\int_{x_0}^{x} a(t)dt\right)$$

c. Verify Abel's formula for the differential equation $y'' - y = 0$ having two solutions $y_1 = e^x$ and $y_2 = e^{-x}$. [2]

OR

a. Describe the undetermined coefficients method to find the general solution of the second-order differential equation $y'' + a\,y' + by = f(x)$ where a and b are constants. [4]

b. Use the undetermined coefficients method to find the general solution of the equation

$$y'' - 2y' - 3y = 4x - 5$$ [3]

2. a. Consider the non-homogeneous linear system of two first-order equations

$$\frac{dx}{dt} = a_{11}\,x + a_{12}\,y + f(t), \qquad \frac{dy}{dt} = a_{21}\,x + a_{22}\,y + g(t)$$

Let (x^*, y^*) be the general solution of this non-homogeneous linear system, and let (x_p, y_p) be any particular solution of this system. Then show that $(x^* - x_p,\ y^* - y_p)$ is the general solution of the associated homogeneous system of this non-homogeneous system. [3]

b. Find the general solution of the non-homogeneous system [4]

$$\frac{dx}{dt} = -x + y + t, \qquad \frac{dy}{dt} = -5x + 3y$$

3. a. Prove that a differential equation of the form $M(x, y)\,dx + N(x, y)\,dy = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. [4]

b. Show that the differential equation
$(y\sin x + \sin y + y)\,dx + (-\cos x + x\cos y + x)\,dy = 0$ is exact, and then solve it. [3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. A 12 volt battery is connected to a series circuit in which the inductance is $\frac{1}{2}$ henry and the resistance is 10 ohms. Determine the current I if the initial current is zero.

OR

The population of bacteria in a culture grows at a rate proportional to the number of bacteria present at time t. After 3 hours it is observed that 400 bacteria are present. After 10 hours 2000 bacteria are present. What was the initial number of bacteria?

5. Use D'Alembert's method to find the solution $u(x, t)$ of 1D wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u}{\partial t}$.

6. Use the determinant method to find the solution $(x(t),\ y(t))$ of the homogeneous linear system of equations

$$\frac{dx}{dt} = 3x - 5y, \qquad \frac{dy}{dt} = 4x + 8y$$

7. Use a variation of the parameter method to find the unique solution $y(x)$ of the initial value problem

$$y'' - 3y' + 2y = 10 \sin x, \qquad y(0) = 1, \ \ y'(0) = -6$$

8. Using separating variables, find the solution $u(x, y)$ of the partial differential equation $y^2 u_x - x^2\, u_y = 0$.

9. Show that the linear first-order differential equation $\frac{dy}{dx} + p(x)\, y = q(x)$ is not exact. Find its integrating factor, and show that its solution is

$$y(x) = \left[\int \left(q(x) \exp(\textstyle\int p(x)dx)\right) dx + c\right] \exp(-\textstyle\int p(x)dx)$$

where c is a constant.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Solve the Euler-Cauchy equation $x^2 y'' - 4x\, y' + 6y = 0$.

11. Use the variable separation method to solve $y' \sin x - y \cos x = 0$.

12. Transform the equation $x''' - 3x'' + 4x' - x = 0$, where $' \equiv \frac{d}{dt}$ into a system of first-order equations.

13. Solve the partial differential equation $u_{xy} = u_x$ using the method treating PDE like an ODE.

14. Find the general solution of $y = 2xp - p^2$ where $p \equiv \frac{dy}{dx}$.