KATHMANDU UNIVERSITY
End Semester Examination
January/February 2024

15 FEB 2024

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 217
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date :

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The second-order ordinary differential equation in $x(t)$ of two first-order linear system $\frac{dx}{dt} = y, \ \frac{dy}{dt} = -3t^3x + 6ty$ is ________________.

2. The integrating factor of the differential equation $\frac{dy}{dx} = -\frac{3}{x}y + e^{-x}$ is ____________.

3. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + \cos\left(\frac{y}{x}\right) = 0$ is ___________.

4. Suppose $X_1 = (x_1, y_1)$ and $X_2 = (x_2, y_2)$ are two solutions of the homogeneous system $X' = AX$, then their Wronsian is ________________.

5. The type of the partial differential equation $x^2u_{xx} + 2y\,u_{xy} - u_{yy} = 0$ is ________________ when $(x, y) \neq (0,0)$.

6. The solution $u(x, y)$ of the partial differential equation $u_x = 0$ with $u(0, y) = \cos y$ is $u(x, y) =$ ________________.

7. A first-order differential equation $\frac{dy}{dx} = -\frac{M(x,y)}{N(x,y)}$ is exact if ________________.

8. The function $y(x) = a\,e^{2x} + b\,e^{5x}$ where a and b are constants is a general solution of the second-order homogeneous differential equation ________________.

9. The solution of the initial value problem $\frac{dy}{dx} = 100\,x, \ \ y(0) = 1$ is $y(x) =$ ___________.

10. Two solutions y_1 and y_2 of a second-order homogeneous differential equation are linearly dependent if $\frac{y_1}{y_2} =$ ________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK THE ANSWER**, by selecting the most appropriate answer from among the given ones.

11. The order of the differential equation $\sin^2 x \frac{d^2y}{dx^2} + y^2 \left(\frac{dy}{dx}\right)^2 = e^{x+y}$ is ___________.
    [0; 1; 2; 4]

12. The solution of the differential equation $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ is ____________________, where c is a constant.
    [$x^2 - y^2 = cx$; $x^2 + y^2 = cy$; $x^2 + y^2 = cx$; $x^2 - y^2 = cy$]

13. Consider a homogeneous system of two linear differential equations $\frac{dx}{dt} = ax + by$, $\frac{dy}{dt} = cx + dy$ and suppose (x_1, y_1) and (x_2, y_2) be any two solutions of this homogeneous equation on an interval I. Then these two solutions are linearly independent in I if and only if their Wronskian $W(t)$ ____________________.
    [equals to zero; equals to a non-zero; is not defined; is defined only for $t > 0$]

14. The initial value problem $X' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} X + \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{4t}$, $X(0) = \frac{1}{5}\begin{bmatrix} 3 \\ 22 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $X' = \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix}$ has a unique solution on ____________________.
    [$(-\infty, 0)$; $(0, \infty)$; $[0, \infty)$; $(-\infty, \infty)$]

15. The model equation for a two-dimensional wave equation is ____________.
    [$u_t = u_{xx}$; $u_t = u_{xx} + u_{yy}$; $u_{tt} = u_{xx}$; $u_{tt} = u_{xx} + u_{yy}$]

16. The Bernoulli differential equation $\frac{dy}{dx} + p(x)y = q(x)\, y^n$ is a linear homogeneous if $n =$ ______.
    [-1; 0; 1; $\frac{1}{2}$]

17. Hyperbolic equations of a second-order partial differential equation have ___________ families of characteristic curves.
    [three; one; no; two]

18. Roots of the characteristic (auxiliary) equation of the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 4\, e^x$ are ____________________.
    [real and equal; real and unequal; complex and equal; complex and unequal]

19. A second-order differential equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ is a ___________ equation.
    [Lagrange; Cauchy; Clairaut; Maclaurin]

20. A partial differential equation has ____________________ variable(s).
    [one independent; two or more independent; more than one dependent; equal number of dependent and independent]

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F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. a. Suppose $(x_1(t),\ y_1(t))$ and $(x_2(t),\ y_2(t))$ are solutions of the homogeneous system

$$\frac{dx}{dt} = ax + by, \qquad \frac{dy}{dt} = cx + dy$$

on an interval I. Then, prove that these two solutions are linearly independent on an interval I if, and only if their Wronskian $W(t) \neq 0$ in I. [4]

b. Rewrite the system

$$\frac{dx}{dt} = 2x + 4y, \qquad \frac{dy}{dt} = 4x + 2y$$

in matrix form, and verify that the vector function $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{6t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t}$ satisfy the system for any choice of the constants c_1 and c_2. [3]

2. a. Discuss step-by-step procedures to transform a second-order partial differential equation

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$$

into normal form. [3]

b. Find the type, transform to normal form, and solve $u_{xx} + 2\,u_{xy} + u_{yy} = 0$. [4]

3. a. If $y_h(x)$ is the general solution of the associated homogeneous equation of the non-homogeneous differential equation

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$$

and $y_p(x)$ be its particular integral. Then, prove that $y_h(x) + y_p(x)$ is also its solution. [3]

b. Use the variation of parameters method to derive a relation for a particular integral of the non-homogeneous ordinary differential equation in (a). [4]

OR

a. If $M(x,y)dx + N(x,y)dy = 0$ is a non-exact differential equation and $F(x)$ is its integrating factor, then prove that $F(x) = \exp(\int \phi(x)dx)$ where $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \phi(x)$. [3]

b. Find the integrating factor of the differential equation $\frac{dy}{dx} + p(x)y = q(x)$ and show that its solution is

$$y(x) = \left[\int q(x) \exp\left(\int p(x)dx\right) dx + C\right] \exp\left(-\int p(x)dx\right)$$

where c is a constant. [4]

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SECTION "D"

[6 Q. × 4 = 24 marks]

4. Use the elimination method to solve the system of two linear differential equations

$$\frac{dx}{dt} = x + 2y, \qquad \frac{dy}{dt} = 3x + 2y$$

5. Solve the Bernoulli equation $\frac{dy}{dx} - 2xy = 2x\,y^2,\ \ y(0) = 1.$

OR

Solve the linear first-order differential equation $\frac{dy}{dx} + y = x\,e^{-x},\ \ y(0) = 1.$

6. Model the RL-circuit, and solve the resulting ordinary differential equation for t when $R = 11$ ohms, $L = 0.1$ henrys, and $E(t) = 48$ volts.

7. Find the solution $u(x, y)$ of the partial differential equation $u_{xx} - u_x - 2u = 3\,e^{2x}$ using solvable as ordinary differential equation method.

8. Solve the non-homogeneous Euler-Cauchy equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3 \sin x$ using the variation of parameters method.

9. Use the method of undetermined coefficients to solve the initial value problem

$$\frac{d^2y}{dx^2} + y = 0.001\,x^2, \qquad y(0) = 0,\ y'(0) = 1.5.$$

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Solve the non-linear differential equation $p^2 - 8p + 15 = 0$ where $p = \frac{dy}{dx}$.

11. The D'Alembert's solution $u(x,t)$ of the 1D wave equation $u_{tt} = c^2u_{xx},\ 0 < x < L$ subjected to initial conditions $u(x, 0) = f(x),\ u_t(x, 0) = 0$ and boundary conditions $u(0,t) = 0,\ u(L,t) = 0$ is given by $u(x,t) = \frac{1}{2}[f(x + ct) + f(x - ct)]$. Assume that $c = 5$ meters/second and $L = 6$ meters. Find the value of $u(2, 2)$ when

$$f(x) = \begin{cases} 0.01\,x^2, & 0 \le x \le 3 \\ 0.01\,(6 - x)^2, & 3 \le x \le 6 \end{cases}$$

12. Find the general solution of the ordinary differential equation $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0.$

13. Use reduction into variable separation method to solve $\frac{dy}{dx} = (x + y)^2.$

14. The population of a community is known to increase at a rate proportional to the number of people present at time t. If an initial population P_0 has doubled in 5 years, how long will it take to triple?