KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2025

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 217
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 02 MAR 2025

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. The differential equation of the family of curves $y = c\,e^{5x}$ is ____________________, where c is a constant.

2. The degree of the differential equation $x\frac{dy}{dx} = \sqrt{y}\left(\frac{dy}{dx}\right)^2 + \sin x$ is ________________.

3. In a closed electric circuit if R is the resistance of a resistor, L is the inductance of an inductor, and $E(t)$ is the e.m.f, then the model equation for current in the RL-circuit is ____________________________.

4. The partial differential equation $u_{xx} + 4\,u_{yy} = 0$ is of type ____________________.

5. The solution $u(x,y)$ of the partial differential equation $u_x = 0,\ u(0,y) = e^y$ is $u(x,y)$ = ________________.

6. The second-order ordinary differential equation $y'' - 3y' + 2y = 0$ has the general solution $y(x)$ = _________________________, where c_1 and c_2 are arbitrary constants.

7. The integrating factor of the ordinary differential equation $y' + \frac{1}{x}\,y = f(x)$ is ________.

8. The second-order ordinary differential equation in $x(t)$ of the system $\frac{dx}{dt} = x - y,\ \frac{dy}{dt} = x + y$ is _________________________________.

9. A differential equation $\frac{dy}{dx} = -\frac{M(x,y)}{N(x,y)}$ is exact if _____________________________.

10. The solution $u(x,t)$ of 1D wave equation $u_{tt} = c^2 u_{xx},\ 0 < x < L$ subjected to boundary conditions $u(0,t) = u(L,t) = 0$, and initial conditions $u(x,0) = f(x),\ u_t(x,0) = 0$ is given by $u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)]$. If $c = 5$ meter per second, $L = 6$ meter, and $f(x) = 0.1\sin(2\pi x/L),\ 0 < x < L$. Then $u(2,2)$ = ________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (***DO NOT TICK THE ANSWER***)

11. The number of arbitrary constants in the particular solution of a third-order differential equation is ______________.
[3; 2; 1; 0]

12. Suppose the characteristic equation of the differential equation $y'' + ay' + by = f(x)$ has complex roots $\alpha \pm i\,\beta$. Then the complementary function $y_c(x) =$ ______________, where A and B are arbitrary constants.
[$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$; $e^{\alpha x}\,(A\sin\beta x + B\cos\beta x)$; $A\,e^{(\alpha+i\beta)x} + Be^{(\alpha-i\beta)x}$; All]

13. The solution of the differential equation $y - x\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2$ is ______________, where c is a constant.
[$y + cx = c$; $y - cx = c$; $y + cx = c^2$; $y - cx = c^2$]

14. The partial differential equation $u_{xx} + u_{yy} = 0$ is known as ______________ equation.
[heat; wave; Laplace; Poisson]

15. The second order linear partial differential equation $-x\,u_{xx} + 2y\,u_{xy} + x\,u_{yy} = 0$ is ______________ at origin.
[parabolic; elliptic; hyperbolic; mixed type]

16. Suppose (x_1, y_1) and (x_2, y_2) are any two linearly independent solutions of a 2×2 homogeneous system $X' = AX$. Then the Wronskian $W(t)$ of these two solutions is ______________ for all t.
[zero; not zero; greater than zero; less than zero]

17. The solution of the system of linear differential equation $\frac{dx}{dt} = y,\ \ \frac{dy}{dt} = x$ is ______________.
[$(c_1e^{-t} + c_2e^{t},\ c_1e^{-t} + c_2e^{t})$; $(c_1e^{-t} + c_2e^{t},\ c_1e^{-t} - c_2e^{t})$; $(c_1e^{-t} + c_2e^{t},\ -c_1e^{-t} + c_2e^{t})$; $(c_1e^{-t} - c_2e^{t},\ -c_1e^{-t} - c_2e^{t})$]

18. The solution of the model $\frac{dT}{dt} = k\,(T - T_0),\ k < 0$ approaches ________ when $t \to \infty$.
[0; ∞; T_0; k]

19. A differential equation of the form $y = x\,f(p) + g(p)$ where $p = \frac{dy}{dx}$ is known as ______.
[Euler; Burger; Clairaut; Lagrange]

20. One of the solutions of $x^2y'' - 3xy' + 4y = 0$ is ______________.
[e^{2x}; $x\,e^{2x}$; $x^{-2}\ln x$; $x^2\ln x$]

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Time : 2 hrs. 30mins.		F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1.

a. Show that a non-homogeneous linear first-order differential equation $\frac{dy}{dx} + p(x)y = q(x)$ is non-exact, and so find its integrating factor, and then show that its general solution is given by

$$y(x) = e^{-\int p(x)dx} \left[\int \left(q(x)\, e^{\int p(x)dx}\right) dx + C\right]$$

where C is an arbitrary constant. [5]

b. Use the above method to find the general solution of the linear first-order differential equation
$\frac{dy}{dx} - y = x.$ [2]

OR

a. Prove that a differential equation of the form $M(x,y)dx + N(x,y)dy = 0$ is exact if, and only if
$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$ [4]

b. Determine whether the differential equation $(x^2 + y^2)\, dx - 2xy\, dy = 0$ is exact or not. If yes, solve it, otherwise find its integrating factor and then solve it. [3]

2.

a. Show that the particular integral $y_p(x)$ of the second order differential equation $\frac{d^2y}{dx^2} + a(x)\frac{dy}{dx} + b(x)\, y = f(x)$ where $a(x), b(x)$ and $f(x)$ are arbitrary functions that are continuous on some interval I, under the method of variation of parameters is given by

$$y_p(x) = -y_1(x) \int \frac{y_2(x)\, f(x)}{W(x)}\, dx + y_2(x) \int \frac{y_1(x)\, f(x)}{W(x)} dx$$

where $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of the associated homogeneous equation $\frac{d^2y}{dx^2} + a(x)\frac{dy}{dx} + b(x)\, y = 0$ and $W(x)$ is the Wronskian of $y_1(x)$ and $y_2(x)$. [4]

b. Solve the non-homogeneous second-order differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x$ determining the particular integral using the variation of parameters method. [3]

P.T.O.

3.

a. Discuss the elimination method to find the solution of the linear homogeneous system [4]

$$\frac{dx}{dt} = a_{11}x + a_{12}y, \qquad \frac{dy}{dt} = a_{21}x + a_{22}y$$

b. Use the elimination method to find the solution of the homogeneous system [3]

$$\frac{dx}{dt} = 3x - y, \qquad \frac{dy}{dt} = 6x - 4y$$

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Reduce the 1D wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ into normal form and solve it.

OR

Find the solution $u(x, y)$ of the partial differential equation $\frac{\partial^2 u}{\partial x^2} - 3\frac{\partial u}{\partial x} - 10\,u = 7\,e^{3x}$.

5. Solve the non-homogeneous system of linear differential equations
$\frac{dx}{dt} = 3x + y - 5\,e^t, \quad \frac{dy}{dt} = -x + y + e^t$

6. Use the method of undetermined coefficients to solve
$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4\,y = 6xe^{3x}, \; y(0) = 0, y'(0) = 3$

7. A large tank is filled to a capacity with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at the rate of 5 gallons per minute. The well-mixed solution is pumped out at the same rate.
   a. Find the model of the problem.
   b. Find the amount of the salt in the tank at time t.

8. Reduce the Bernoulli equation $\frac{dy}{dx} + p(x)\,y = q(x)\,y^n, \; n \neq 0, 1$ into a linear first order differential equation, and hence use it to solve the differential equation $\frac{dy}{dx} - 2xy = 2xy^2$.

9. Find the solution of the Euler-Cauchy differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = 2\,x^3$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the solution $u(x, y)$ of the partial differential equation $y^2u_x - x^2y_y = 0$ using separation of variables method.

11. Solve the non-linear ordinary differential equation $p^2 - 3\,p + 2 = 0$ where $p = \frac{dy}{dx}$.

12. Find the solution $y(x)$ of the differential equation $\frac{dy}{dx}\sin x - y\cos x = 0$.

13. Classify the partial differential equation $u_{xx} + x\,u_{yy} = 0$.

14. Show that if the two functions $y_1(x)$ and $y_2(x)$ are linearly dependent, then their Wronskian equals zero.