KATHMANDU UNIVERSITY
End Semester Examination
May/June, 2022

Marks Scored:

Level : B.Sc.
Year : II
Exam Roll No. :

Registration No.:

Time: 30 mins.

Course : MATH 213
Semester : II
F. M. : 20

Date :

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. If a real number is not rational then it is ______________.

2. A set A is said to be countable if there exists a function $f : A \to \mathbb{N}$ such that f is ______________.

3. If f is real valued and monotonic on $[a, b]$ then f is ______________.

4. A convergent sequence of real numbers converges to $a(n)$ ____________ limit.

5. The set of negative integers is ______________ above by 0.

6. If a sequence is unbounded or it does not converge then this sequence is called a ______________________ sequence.

7. If f is differentiable in $[a, b]$ then it is monotonically increasing if $f'(x)$ ________ 0.

8. The value of $\lim_{x\to\infty} \frac{\ln x}{x^5}$ is ______________.

9. If $I = [0, 8]$ and $P = (0, 1, 3, 8)$, the norm of partition of P is equal to__________.

10. If the function $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x + 5$, then $f^{-1}(0) =$ __________.

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. A number which is neither even nor odd is __________________.
[0; 2π;
$2n$ such that $n \in \mathbb{N}$; 5]

12. Every pair of real numbers a and b satisfy the following conditions $a > b, a = b, a < b$. This property is known as __________________.
[commutative law; associative law; transitive law; trichotomy law]

13. A convergent sequence has only ______________ limit(s).
[one; two; three; less than three]

14. If two sub-sequences of a sequence converge to two different limits, then the sequence is ____________________.
[may convergent; may divergent; convergent; divergent]

15. Every Cauchy sequence has a ________________________________.
[convergent subsequence; increasing subsequence ; decreasing subsequence. positive subsequence]

16. Let $\sum a_n$ be a series of non-negative terms. Then it is convergent if ________________.
[increasing; decreasing; bounded; unbounded]

17. Which of the following statement is **NOT** correct? ____.
[Every increasing sequence of positive numbers diverges or has single limit point;
Every bounded and infinite sequence of real numbers has at least one limit point;
Every monotonic real number sequence is convergent;
Every convergent real number sequence is bounded]

18. If $f'(x)$ exists then $f(x)$ is $a(n)$ ______________ function if $f'(x) = 0$.
[constant; $f'(x)$ increasing; $f'(x)$ bounded; decreasing]

19. If f is differentiable at x_0 in $[a, b]$ then f is ________________ at x_0.
[infinite; bounded; continuous; discontinuous]

20. If a function is Riemann integrable on $[a, b]$ then function must be ______________.
[continuous on $[a, b]$; undefined on $[a, b]$; monotone on $[a, b]$; differentiable on $[a, b]$]

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Level : B.Sc.	Course : MATH 213
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Time : 2 hrs. 30 mins.	F.M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Define lower bound and upper bound of a set of real numbers. Prove that the intersection of finite collection of open sets of real numbers is open. [2 + 5]

2. Define accumulation point and adherent point of a set of real number. State and prove Bolzano–Weierstrass theorem. [2 + 5]

OR

Define the convergent series. Let (x_n) be a sequence of nonnegative real numbers. Then prove that the series Σx_n converges if and only if the sequence $S = (s_n)$ of partial sums is bounded. Test whether the series $\sum \frac{n!n!}{(2n)!}$ converges or not. [1+3+3]

3. Prove that if $f : I \to \mathbb{R}$ has a derivative at $c \in I$, then f is continuous at c but converse may not be true. Evaluate : $\lim_{x \to 0} \sin x^{2 \tan x}$ [2+2+3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. State and prove the Cantor's theorem.

5. Define the convergent sequence. Prove that a sequence in $\mathbb{R}$ can have at most one limit.

6. Prove that an upper bound u of a non-empty set S in $\mathbb{R}$ is the Supremum of S if and only if every $\varepsilon > 0$, there exist an element $s \in S$ such that $u - \varepsilon < s$

7. State and prove the Squeeze theorem.

8. State and prove Cauchy Mean Value Theorem.

OR

Define derivative of a function at a point. If $f, g : I \to \mathbb{R}$ has a derivative at $c \in I$, then prove that f is continuous at c and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.

9. Define Riemann integrable function. Prove that if $f \in R[a, b]$, then the value of integral of f is uniquely determined.

SECTION "E"
[5 Q. × 2 = 10 marks]

10. Let $A = \{x \in \mathbb{R} : x \neq 5\}$ and define $f(x) = \frac{5x}{x-5}$ for all x in A. Show that f is one to one.

11. Determine the set $A = \{x \in \mathbb{R} : |x-1| < |x|\}$.

12. Prove that a convergent sequence of real numbers is bounded.

13. Define the norm of partition. If $I = [0, 7]$, calculate the norm of

$$P = (0, 0.5, 2.5, 3, 3.5, 4, 5, 7)$$

14. If $f: I \to \mathbb{R}$ is bounded and P is any partition of I, prove that $L(f; P) \leq U(f; P)$.