KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 213
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 21 JUL 2023

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. A __________ subset of the set ℝ of real numbers is an interval.
2. A set A is __________ if there exists a function $f : A \to \mathbb{N}$ such that f is a bijection.
3. A real valued and monotonic function f on $[a, b]$ is ______________.
4. A convergent sequence of real numbers has __________ limit point.
5. The sum of rational and irrational numbers is ________________.
6. The ____________ of finite collection of closed sets in ℝ is closed.
7. The property that states that "Every pair of real numbers a and b satisfied the following conditions: $a > b$, or $a = b$, or $a < b$", is known as ___________________.
8. The sum of first five terms of the sequence $x_n = \frac{(-1)^n}{n}$ is ______________.
9. The end points of the image interval are the ___________ extreme values of the function.
10. The set of integers has no ___________ point.

SECTION "B"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by choosing the most appropriate answer from the given ones.

11. A number which is neither even nor odd is ______________.
[0; 2π; $2n$ for $n \in \mathbb{N}$; 5]

12. The series $1 + r + r^2 + r^3 + \cdots$ is oscillatory if __________.
[$r = -1$; $r < 1$; $r > 1$; $r = 1$]

13. The infimum of the set $\{0.1, 0.16, 0.166, 0.1666, \ldots\}$ is ______________.
[0; 0.1; 0.2; 0.3]

14. The solution set for $B = \{x \in \mathbb{R}: |x - 1| < |x|\}$ is ________________.
[(1/3, ∞); (1/2, ∞); (1, ∞); (-∞, ½)]

15. Every Cauchy sequence has a ____________ subsequence.
[convergent; increasing; decreasing; positive]

16. The series $\sum a_n$ of non-negative terms is convergent if it is ________________.
[increasing; decreasing; bounded; unbounded]

17. The third term of the 3-tail of the sequence $X = (2, 4, 6, 8, 10, 12, \dots, 2n, \dots)$ is ______.
[10; 12; 14; 16]

18. For the real function $g(x) = 4x + 3$, $g^{-1}(x)$ is ____________.
[$\frac{x+3}{4}$; $\frac{x-3}{4}$; 1; 7]

19. The norm of the partition $P = (0, \frac{1}{4}, \frac{3}{4}, 1)$ of $[0, 1]$ is ____________.
[-1; $-\frac{1}{2}$; $\frac{1}{2}$; 1]

20. A Riemann integrable function defined on $[a, b]$ is __________ on $[a, b]$.
[continuous; bounded; monotone; derivable]

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Semester : II
F.M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. What is meant by a bounded subset of the set $\mathbb{R}$ of real numbers ? State the completeness property of $\mathbb{R}$ relative. Also, use it to prove that $inf\ (A + B) = inf\ A + inf\ B$, where $A + B = \{a + b : a \in A, b \in B\}$, A and B being bounded non-empty subsets of $\mathbb{R}$. [1+2+4]

2. Define a Cauchy sequence with example. Also, state and prove the Cauchy convergence criteria for the sequence of real numbers. [2+1+4]

OR

Discuss the convergence or divergence of an infinite series generated by a sequence in $\mathbb{R}$ with example. Also, test the convergence of the 2-series $\sum \frac{1}{n^2}$. [2+1+3]

3. State Lagrange mean value theorem and write its geometrical interpretation. Also, verify it for $f(x) = x^2 + 2x + 3$ on $[0, 3]$. [2+2+3]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Prove that the union of any arbitrary family of open set of real numbers is open.
5. Evaluate $\lim_{x \to 2} x^2 + 2$ and verify your result.
6. State and prove the Squeeze theorem for the sequence of real numbers.
7. State and prove the product rule for two differentiable functions on a set.
8. Define a Riemann integrable function. Prove that, if $f \in \mathcal{R}[a, b]$, then the value of integral of f is uniquely determined, where symbol has usual meaning.

OR

Evaluate $\int_0^2 x^2\ dx$ as the limit of sum and also verify the result.

9. For all real values, solve the inequality: $|x - 3| + |x + 1| \leq 6$.

SECTION "E"
[5Q. × 2 = 10 marks]

10. If for $a, b \in \mathbb{R}$ and for each $\varepsilon > 0, a \leq b + \varepsilon$, then prove that $a \leq b$.
11. Prove that $|ab| = |a||b|$ for all $a, b \in \mathbb{R}$.
12. Prove that a set A is closed if and only if $A = \bar{A}$.
13. Evaluate: $\lim_{x \to 0^+} x^x$.
14. Verify the uniform continuity of $f(x) = x^2$ on $[0, a]$.