KATHMANDU UNIVERSITY
End Semester Examination
January/February 2024

Level : B.Sc.
Year : II
Time : 2 hrs. 30mins.

08 JUL 2024

Course : MATH 213
Semester : II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define suprimum and infimum of a set of real numbers. Prove that the intersection of finite collection of open set of real numbers is open and union of finite collection of closed set of real numbers is closed. [2+3+2]

2. Define interior point and adherent point of a set of real number. State and prove Bolzano–weierstrass theorem. [2+5]

OR

Define the convergent sequence. Let (x_n) be a sequence of nonnegative real numbers. Then the series $\Sigma\ x_n$ converges if and only if the sequence S = (s_n) of partial sums is bounded. Test whether the series $\sum \frac{1}{n^2+n}$ is converges or not. [1+3+3]

3. Define the limit of a function. Let functions f and g be define on [a, b], differentiable at a, *g'(a)≠ 0, f(x) = g(x)=0 and g(x) ≠ 0 for a < x <b* . then prove that the limit of (f/g) at *a* exact and is equal to f'(a)/g'(a) i.e.
$\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{f'(a)}$ and evaluate : $\lim\limits_{x\to 0} sinx^{2tanx}$ [1+2+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. State and prove the cantor's theorem.

5. If $f: I \to \Re$ has a derivative at $c \in I$, then f is continuous at c but converse may not be true.

6. Prove that an upper bound u of a non-empty set S in $\Re$ is the Suprimum of S if and only if every $\varepsilon > 0$, there exist an element $s \in S$ such that $u - \varepsilon < s$

7. Define the Cauchy sequence. Prove that every convergent sequence of real numbers is a Cauchy sequence.

8. Define Riemann integrable function. Prove that, if $f \in R[a, b]$, then the value of integral of f is uniquely determined.

P.T.O.

9. Define derivative of a function at a point. If $f, g : I \to \Re$ has a derivative at $c \in I$, then prove that f is continuous at c and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.

OR

Define the continuous function on real numbers. State and prove Bolzano's intermediate value theorem.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Let $A = \{x \in \Re : x \neq 3\}$ and define $f(x) = \frac{5x}{x-3}$ for all x in A. Show that f is one to one.

11. If a, b $\in \Re$, prove that $|a + b| \leq |a| + |b|$

12. Prove that a sequence in $\Re$ can have at most one limit.

13. Verify the Lagrange mean value theorem for $f(x) = x^2 + 2x + 5$ on [2, 7].

14. If $f: = I \to \Re$ is bounded and P is any partition of I, prove that $L(f; P) \leq U(f; P)$