KATHMANDU UNIVERSITY
End Semester Examination
January/February 2024

Marks Scored:

Level : B.Sc.
Year : II

09 FEB 2024

Course : MATH 213
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date :

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The function f is said to be____________________ if whenever $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.
2. A set A is said to be ____________________ if there exists a function $f : A \to \mathbb{N}$ such that f is bijection.
3. A set S is said to be countable if it is either finite or ____________________.
4. A set is said to be bounded if it is both bounded above and __________________.
5. If a sequence has no limit, we say that the sequence is __________________.
6. A sequence $X = (x_n)$ of real numbers is said to be bounded if there exists a real number $M > 0$ such that __________________ M for all $n \in \mathbb{N}$.
7. If f is differentiable in $[a, b]$ then it is monotonically increasing if $f'(x)$______0.
8. The value of is $\lim_{x\to\infty} \frac{\ln x}{x}$__________________
9. If $I = [0, 8]$ and partition $P = (0, 1, 4, 6, 8)$, the norm of partition of P is equal to__________________
10. If $f'(x)$ exists and $f(x)$ is __________________ function then $f'(x)=0$.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones

11. Which of the following number is not rational ____________?
    [0; π; 2n such that $n \in \mathbb{N}$; 5.75]
12. Every pair of real numbers a and b satisfied the following conditions $a > b$, $a = b$, $a < b$. This property known as __________________
    [Commutative Law; Associative Law; Transitive Law; Trichotomy Law]

13. If a sequence has no limit, we say that the sequence is ________________

[bounded; unbounded; convergent; divergent]

14. If two sub-sequences of a sequence converge to two different limits, then the sequence is ________________

[may convergent; may divergent; convergent; divergent]

15. Every Cauchy sequence has a ____________________.

[convergent subsequence; increasing subsequence; decreasing subsequence; positive subsequence]

16. Let $\sum a_n$ be a series of non-negative terms. Then it is convergent if ___________

[increasing; decreasing; bounded; unbounded]

17. Which of the following statements is **NOT CORRECT**?

____

[Every increasing sequence of positive numbers diverges or has single limit point;
Every bounded and infinite sequence of real numbers has at least one limit point;
Every monotonic real number sequence is convergent;
Every convergent real number sequence is bounded]

18. If the function f : $\Re \rightarrow \Re$ be defined by $f(x) = x + 5$, then $fof^{-1}(x) =$ __________.

[x; 2x; -x; 1/x]

19. If f is differentiable at x_0 in [a, b] then f is ________________ at x_0.

[infinite; bounded; continuous; discontinuous]

20. Let I = [a, b] and let $f : I \rightarrow \Re$ be a bounded function. Then f is said to be Darboux integrable on I if (f) ______ $U(f)$.

[$\leq$; $<$; $\geq$; $=$]

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08 JUL 2024

Course : MATH 213
Semester : II
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Define suprimum and infimum of a set of real numbers. Prove that the intersection of finite collection of open set of real numbers is open and union of finite collection of closed set of real numbers is closed. [2+3+2]

2. Define interior point and adherent point of a set of real number. State and prove Bolzano–weierstrass theorem. [2+5]

OR

Define the convergent sequence. Let (x_n) be a sequence of nonnegative real numbers. Then the series $\Sigma\ x_n$ converges if and only if the sequence $S = (s_n)$ of partial sums is bounded. Test whether the series $\sum\frac{1}{n^2+n}$ is converges or not. [1+3+3]

3. Define the limit of a function. Let functions f and g be define on [a, b], differentiable at a, *g'(a)≠ 0, f(x) = g(x)=0 and g(x) ≠ 0 for a < x <b* . then prove that the limit of (f/g) at *a* exact and is equal to f'(a)/g'(a) i.e.
$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f'(a)}{f'(a)}$ and evaluate : $\lim_{x\to 0} sinx^{2tanx}$ [1+2+3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. State and prove the cantor's theorem.

5. If $f : I \to \Re$ has a derivative at $c \in I$, then f is continuous at c but converse may not be true.

6. Prove that an upper bound u of a non-empty set S in $\Re$ is the Suprimum of S if and only if every $\varepsilon > 0$, there exist an element $s \in S$ such that $u - \varepsilon < s$

7. Define the Cauchy sequence. Prove that every convergent sequence of real numbers is a Cauchy sequence.

8. Define Riemann integrable function. Prove that, if $f \in R[a, b]$, then the value of integral of f is uniquely determined.

P.T.O.

9. Define derivative of a function at a point. If f, $g : I \to \Re$ has a derivative at $c \in I$, then prove that f is continuous at c and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.

OR

Define the continuous function on real numbers. State and prove Bolzano's intermediate value theorem.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Let $A = \{x \in \Re : x \neq 3\}$ and define $f(x) = \frac{5x}{x-3}$ for all x in A. Show that f is one to one.

11. If $a, b \in \Re$, prove that $|a + b| \leq |a| + |b|$

12. Prove that a sequence in $\Re$ can have at most one limit.

13. Verify the Lagrange mean value theorem for $f(x) = x^2 + 2x + 5$ on $[2, 7]$.

14. If $f: = I \to \Re$ is bounded and P is any partition of I, prove that $L(f;\ P) \leq U(f;\ P)$