KATHMANDU UNIVERSITY
End Semester Examination
February, 2025

Marks Scored:

Level : B.Sc.		Course : MATH 213
Year : II		Semester : II
Exam Roll No. :	Time: 30 mins.	F. M. : 20
Registration No.:		Date : 24 FEB 2025

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. $a + b = b + a$ for all a, b in $\Re$, It is called ____________ property of addition.
2. A set A is said to be ____________ if there exists a bijection function $f : A \to N$.
3. The value of $x \in \Re$ such that $|2x + 3| < 7$, ____________
4. If two sub-sequences of a sequence converge to two different limits, then the sequence is ____________
5. The infimum of set of non-negative real number is ____________.
6. A sequence (x_n) of real numbers is said to be bounded if there exists a real number $M > 0$ such that $|x_n|$ ________ M for all $n \in \mathbb{N}$.
7. If f is differentiable in [a, b] and it has local minima at x = c then $f'(c)$ ______ 0.
8. The value of $\lim_{\infty} \frac{x^n}{e^x}$ is ____________
9. If $I = [0, 10]$ and $P = (0, 1, 3, 4, 5, 7.5, 8)$, the norm of partition of P is equal to ____________
10. If the function $f : \Re \to \Re$ be defined by $f(x) = 2x^3 + 500$, then $f \circ f^{-1}(a) =$ ____________

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (***DO NOT TICK THE ANSWER***)

11. Irrational number of following is ____________________
[22/7; π; 7n such that $n \in \mathbb{N}$; 0.33333]

12. Every pair of real numbers a and b satisfied the following conditions $a > b$, $a = b$, $a < b$. This property known as ____________________
[Commutative Law; Associative Law; Transitive Law; Trichotomy Law]

13. A convergent sequence has only ____________________ limit(s).
[one; two; three; No]

14. If a sequence has unique limit, then the sequence is ____________________
[may convergent; may divergent; convergent; divergent]

15. Every convergent sequence is a ____________________
[Cauchy sequence. increasing equence
decreasing sequence. Monotonic sequence]

16. Let $\sum a_n$ be a series of non-negative terms. Then it is convergent if ____________
[Increasing; Decreasing; bounded; unbounded]

17. If $c \in A$ is not a cluster point of A, that point is called isolated point
[Boundary point; Limit point; isolated point; Fixed point]

18. If $f'(x)$ exists and $f(x)$ is ____________________ function then $f'(x) = 0$
[Increasing; Decreasing; bounded; constant]

19. If f is differentiable at x_0 in [a, b] then f is ____________________ at x_0.
[infinite; bounded; continuous; Discontinuous]

20. Let I = [a, b] and let $f : I \rightarrow \Re$ R be a bounded function. Then f is said to be Darboux integrable on I if ____________________
[*L(f)* exist; *U(f)* exist; *L(f)* = *U(f)*; *L(f)* and *U(f)* exists]

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SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define countable set and denumerable set of real numbers. Prove that the intersection of finite collection of open set of real numbers is open and union of finite collection of closed set of real numbers is closed. [2+3+2]

2. Define the convergent sequence. Suppose that X = (x_n), Y = (y_n) and Z = (z_n) are sequences of real numbers such that $x_n \le y_n \le z_n$ for all $n \in N$ and that $\lim(x_n) = \lim(z_n)$. Then Y = (y_n) is convergent and $\lim(x_n) = \lim(y_n) = \lim(z_n)$. Test whether the series $\sum\frac{1}{n^2-n+1}$ is converges or not. [1+3+3]

3. Define the limit of a function. Let functions f and g be define on [a, b], differentiable at a, $g'(a) \neq 0$, $f(x) = g(x) = 0$ and $g(x) \neq 0$ for $a < x < b$. then prove that the limit of (f/g) at a exact and is equal to f'(a)/g'(a) i.e.
$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$ and evaluate : $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{sinx}\right)$ [1+2+3]

OR

State and prove L' Hospital rule and evaluate: $\lim_{x\to 0} tanx.logx$. [1+4+2]

SECTION "D"
[6Q. × 4 = 24 marks]

4. State and prove Bolzano–weierstrass theorem.

5. Let I = [a, b] be a closed bounded interval and let $f : I \to \Re$ be continuous on I. Then f is bounded on I.

6. Prove that an upper bound u of a non-empty set S in $\Re$ is the Suprimum of S if and only if every $\varepsilon > 0$, there exist an element $s \in S$ such that $u - \varepsilon < s$

P.T.O.

7. Define the Cauchy sequence. Prove that every convergent sequence of real numbers is a Cauchy sequence.

8. If f, $g : I \rightarrow \Re$ has a derivative at $c \in I$, then prove that f is continuous at c and $\left(\frac{f}{g}\right)'(c) = \frac{g'(c)f(c) - g(c)f'(c)}{(g(c))^2}$.

OR

Define the continuous function on real numbers. State and prove Bolzano's intermediate value theorem.

9. Define Riemann sum and Riemann integrable function. Prove that, if $f \in R[a, b]$, then the value of integral of f is uniquely determined.

SECTION "E"
[5Q. × 2=10 marks]

10. Let $A = \{x \in \Re : x \neq 5\}$ and define $f(x) = \frac{x}{x-5}$ for all x in A. Show that f is one to one.

11. If A and B are two sets of real numbers, prove that : $\overline{A \cup B} = \bar{A} \cap \bar{B}$

12. Prove that a sequence in $\Re$ can have at most one limit.

13. If $f : I \rightarrow \Re$ has a derivative at $c \in I$, then f is continuous at c.

14. If $f: = I \rightarrow \Re$ is bounded and P is any partition of I, prove that
$L(f; P) \leq U(f; P)$