KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 213
Semester : II

Exam Roll No.: Time: 30 mins. F. M. : 20

Registration No.: Date 30 NOV 2023

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. The derived set of the set of integers is ....

2. A set A is countable if there exists a ....function from A to ℕ.

3. A real valued and monotonic function on closed interval is .... variation.

4. A .... sequence of real numbers has unique limit point.

5. A .... sequence of closed bounded intervals has a common point.

6. The .... of finite collection of closed sets in ℜ is closed.

7. The Trichotomy property divides the set of real numbers into .... distinct types of numbers.

8. The sum of first five terms of the sequence $x_n = (-1)^n/n$ is ....

9. The end points of the image interval are the .... extreme values of the function.

10. A connected .... of the set ℜ of real numbers is an interval.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by choosing the most appropriate answer from the given ones.

11. A Riemann integrable function defined on [a, b] is .... on [a, b].
[continuous; bounded; monotone; derivable]

12. The series $1 + r + r^2 + r^3 + \cdots$ is oscillatory if ....
[$r = -1$; $r < 1$; $r > 1$; $r = 1$]

13. The infimum of the set { 0.1, 0.16, 0.166, 0.1666, 0.16666, ... } is ....
[0; 0.1; 0.2; 0.3]

14. The solution set for $B = \{x \in R: x^2 - 16 < 0\}$ is ....
[(-4, 0); (-4, 4); (0, 4); (0, ∞)]

15. Every Cauchy sequence has a ………………….. subsequence.
[convergent; increasing; decreasing; positive]

16. The series $\sum a_n$ of non-negative terms is convergent if it is ……….
[increasing; decreasing; bounded; unbounded]

17. The fourth term of the 3-tail of the sequence Y = (2, 4, 6, 8, 10, 12, … , 2n, …) is …………...
[10; 12; 14; 16]

18. For the real function g(x) = 4x + 3, $g^{-1}(7)$ is ……………..
[-1; 0; 1; 7]

19. The norm of the partition Q = (0, ¼, ¾, 1) of [0, 1] is ……….
[-1; -1/2; 1/2; 1]

20. A number which is neither even nor odd is ………………
[0; 2π; 2n for n ∈ ℕ; 5]

KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

30 NOV 2023

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 213
Semester : II
F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Define a bounded subset of the set $\Re$ of real numbers with example. State the completeness property of $\Re$ relative to supremum. Also, use it to prove that
sup (A + B) = sup A + sup B, where $A + B = \{a + b : a \in A, b \in B\}$, A and B being bounded non-empty subsets of $\Re$. [2+2 + 3]

2. What is meant by a Cauchy sequence of real numbers. Also, state and prove the Cauchy convergence criteria for the sequence of real numbers. [2+1+ 4]

OR

Discuss the convergence or divergence of an infinite series generated by a sequence in $\Re$ with example. Also, test the convergence of the 3-series $\sum\frac{1}{n^3}$. [2+1+3]

3. State and interpret the Lagrange' mean value theorem. Also, verify this theorem for $f(x) = x^2 - 4x - 3$ on [1, 4]. [2+2+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. State and prove the reverse of Triangle inequality on the set of real numbers.

5. Evaluate $\lim_{x\to 2}(x^2 + 3)$ and verify your result.

6. State and prove the Squeeze theorem for the real functions.

7. State and prove the sum rule for two differentiable functions on a set.

8. Define a Riemann integrable function. Prove that, if $f \in R[a, b]$, then the value of integral of f is uniquely determined, symbol has usual meaning.

OR

Evaluate $\int_0^2 (x^2 + 1)\ dx$ as the limit of sum and also verify the result.

9. For all real values, solve the inequality: $\big||x - 1| + 2\big| \le 4$.

SECTION "E"

[5 Q. × 2 = **10** marks]

10. If a ≥ b and b ≥ a for a, b $\in \mathfrak{R}$ then show that a = b.

11. Prove that a set is closed if its complement is open.

12. Prove that the function $f(x) =$ sin x is continuous for every real values.

13. Evaluate: $\lim\limits_{x \to 0^+} x^x$.

14. Verify the uniform continuity of $f(x) = x^2$ on [0, a].