KATHMANDU UNIVERSITY
End Semester Examination
January/February 2024

Marks Scored:

01 FEB 2024

Level : B.Sc.
Year : II

Course : MATH 211
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date :

## SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. If the particle is moving in a plane curve, the component of acceleration along the normal is ____________.

2. The $(p, r)$ equation of parabola with pole at focus is ______________.

3. If orbit is $\frac{a}{r} = e^{n\theta}$, the law of force is ____________.

4. A body is moving in a straight line OAB with SHM has zero velocity when at points A and B whose distances from O are $a$ and $b$ respectively and has a velocity $v$ when half way between them, the complete period is ______________.

5. The relation between linear velocity $(v)$ and angular velocity $(\dot{\theta})$ is _____________.

6. The phase at any time of a simple harmonic motion is ______________.

7. If three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two is the statement of ______________.

8. If the area under consideration is symmetrical about $x$-axis, the centre of gravity lies on _____________.

9. The relation between the tension $T$ at any point and its ordinate $y$ in catenary is _____________.

10. Let $(\xi, \eta)$ be the point on the hodograph corresponding to the point $(x, y)$ on the path, then $(\xi, \eta) =$____________.

## SECTION "B"

[10Q. × 1=10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. The point, at which the whole weight of the body may be considered to act, is known as ______________________.

    [Centre of mass; Centre of curvature; Centre of gravity; Moment of Inertia]

12. If the tangential and normal accelerations are equal and the tangent rotates with constant angular velocity, the equation of the path is of the form ______________.
[$s = 4a \sin\psi$; $s = A\psi^2 + B\psi + C$; $s = Ae^{\psi} + B$; $s = Ae^{-\psi} + B$]

13. The formula for transverse acceleration is ______________.
[$\ddot{r} - \dot{\theta}^2$; $2\dot{r}\dot{\theta} + r\ddot{\theta}$; $\ddot{r} - r\dot{\theta}^2$; $\ddot{r} + r\dot{\theta}^2$]

14. The differential equation of the Central orbit in pedal form is ______________.
[$P = h^2 p^3 \frac{dp}{dr}$; $r^2\dot{\theta} = h$; $P = \frac{p^3}{h^2}\frac{dp}{dr}$; $P = \frac{h^2}{p^3}\frac{dp}{dr}$]

15. The position of a particle moving in a straight line at any time is $x = \alpha \cos nt + \beta \sin nt$ and it executes a simple harmonic motion, then the amplitude is______________.
[$\alpha$; $\beta$; $\alpha^2 + \beta^2$; $\sqrt{\alpha^2 + \beta^2}$]

16. A system of coplanar forces acting at different points of a rigid body reduces only to a couple if ______________.
[$R = 0, G = 0$; $R = 0, G \neq 0$; $R \neq 0, G = 0$; $R \neq 0, G \neq 0$]

17. Let $(P, p)$ and $(Q, q)$ be any two couples having equal and opposite moments so that ______________.
[$Pp - Qq = 0$; $Pp + Qq = 0$; $Pq - Qp = 0$; $Pq + Qp = 0$]

18. If $G_1$ and $G$ be the algebraic sums of the moments of forces of the given system about the point$(a, b)$ and $(0,0)$ respectively, then ______________.
[$G_1 = G + aR_y - b\,R_x$; $G_1 = G + aR_x$;
$G_1 = G + bR_x - aR_y$; $G_1 = G - bR_x$]

19. For very large value of $x$, a catenary behaves ______________.
[exponential curve; elliptical curve; circular curve; parabolic curve]

20. The center of gravity of a plane area bounded by the curve $x = f(y)$, the axis of y, the lines $y = c$ and $y = d$ is ______________.
[$\bar{x} = \frac{1}{2}\frac{\int_a^b x^2 dx}{\int_a^b x dx}, \bar{y} = \frac{\int_c^d xy\, dx}{\int_c^d x\, dx}$; $\bar{x} = \frac{1}{2}\frac{\int_c^d x^2 dy}{\int_c^d x dy}, \bar{y} = \frac{\int_c^d xy\, dy}{\int_c^d x\, dy}$;
$\bar{x} = \frac{1}{2}\frac{\int_a^b y^2 dx}{\int_a^b y dx}, \bar{y} = \frac{\int_a^b xy\, dx}{\int_a^b x\, dx}$; $\bar{x} = 0, \bar{y} = 0$]

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SECTION "C"
[3 Q. × 7 = 21 marks]

1. Find the expressions for the tangential and normal components of accelerations of a particle moving in a plane curve at any instant. A particle moves in a catenary ($s = c \tan \psi$), the direction of its acceleration at any point makes equal angles with the tangent and the normal to the path at the point. If the speed at the vertex where $\psi = 0$ be $u$, show that the velocity and acceleration at any other point are given by $ue^{\psi}$ and $\frac{\sqrt{2}}{c} u^2 e^{2\psi} \cos^2 \psi$. [4+3]

**OR**

Define the radial and transverse components of velocities. Find the expressions for radial and transverse accelerations of a particle moving in a plane curve. A particle moves along a circle $r = 2a \cos\theta$ in such a way that its accelerations towards the origin is always zero. Prove that $\frac{d^2\theta}{dt^2} = -2 \cot\theta \left(\frac{d\theta}{dt}\right)^2$. [1+4+2]

2. A particle moves in a straight line under attraction towards a fixed point on the line varying inversely as the square of the distance from the fixed point. Discuss the nature of motion. If the acceleration is $\frac{\mu}{x^{5/3}}$, find the time of descent to the centre. [4+3]

3. Derive Cartesian equation of a common catenary. Show that a catenary behaves approximately as a parabola for very small values of $x$. Assuming the intrinsic equation of catenary, prove the relations $y = c \sec\psi$ and $y^2 = s^2 + c^2$. [2+2+3]

SECTION "D"
[6Q. × 4=24 marks]

4. Find the centre of gravity of the area bounded by the parabola $y^2 = 4ax$, the axis of $x$ and the latus rectum.

5. Prove that a system of coplanar forces acting at different points of a rigid body may be reduced to a single force, acting through an arbitrary point, and a couple.

**OR**

Define hodograph. The hodograph of an orbit is a parabola whose ordinate increases uniformly. Show that the orbit is a semi-cubical parabola.

6. If in a simple harmonic motion, $u, v, w$ be the velocities at distances $a, b, c$ from a fixed point on a straight line which is not the centre of force, show that the period $T$ is given by

$$\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$

**OR**

A hyperbola is placed in a vertical plane with its transverse axis horizontal; show that the time of descent down a diameter is least when the conjugate diameter is equal to the distance between the foci.

7. If the central orbit is $r^n = a^n \cos n\theta$ under a force towards the pole, find the law of force.

8. Define sag and span of a catenary. A uniform chain of length $\ell$, is suspended from two points $P$ and $Q$ in the same horizontal line. If the tension at $P$ is $n$ times that the lowest point, Show that the span $PQ$ is $\frac{\ell}{\sqrt{n^2-1}}\log\left(n+\sqrt{n^2-1}\right)$.

9. Forces $P, 2P, 3P$ act along the sides of a triangle formed by the lines $x = 0, y = 0$ and $3x + 4y = 5$. Find the magnitude of the resultant and the equation of its line of action.

## SECTION "E"

[5Q. × 2=10 marks]

10. Show that the time of descent down all chords of a vertical circle from the highest point is same.

11. For a catenary, prove that $\frac{1}{\sec\psi+\tan\psi} = e^{-x/c}$.

12. Prove that the acceleration of a point moving in a curve with uniform speed is $\rho\left(\frac{d\psi}{dt}\right)^2$.

13. Prove that the radius vector is either maximum or minimum at an apse.

14. In a simple harmonic motion of amplitude $a$ and period $T$, prove that $\int_0^T v^2 dt = \frac{2\pi^2 a^2}{T}$.